

# Collusion-Preserving Computation

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## Abstract

In *collusion-free* protocols, subliminal communication is impossible and parties are thus unable to communicate “any information beyond what the protocol allows”. Collusion-free protocols are interesting for several reasons, but have specifically attracted attention because they can be used to reduce trust in game-theoretic mechanisms. Collusion-free protocols are impossible to achieve (in general) when all parties are connected by point-to-point channels, but exist under certain physical assumptions (Lepinski et al., STOC 2005) or in specific network topologies (Alwen et al., Crypto 2008).

We provide a “clean-slate” definition of the stronger notion of collusion *preservation*. Our goals in revisiting the definition are:

- To give a definition with respect to *arbitrary* communication resources (that includes as special cases the communication models from prior work). We can then, in particular, better understand what types of resources enable collusion-preserving protocols.
- To construct protocols that allow no *additional* subliminal communication in the case when parties *can* communicate (a bounded amount of information) via other means. (This property is *not* implied by collusion-freeness.)
- To provide a definition supporting *composition*, so that protocols can be designed in a modular fashion using sub-protocols run among subsets of the parties.

In addition to proposing the definition, we explore implications of our model and show a general feasibility result for collusion-preserving computation of arbitrary functionalities.

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# 1 Introduction

*Subliminal channels* in protocols [38, 39, 40] allow parties to embed “disallowed” communication into the messages of the protocol itself, without being detected. (For example, a party might communicate a bit  $b$  by sending a valid protocol-message with first bit equal to  $b$ .) The existence of subliminal channels is often problematic. In a large-scale distributed computation, for instance, subliminal channels could allow two parties to coordinate their actions (i.e., to collude) even if they may not have been aware of each other in advance. In other settings, parties may be disallowed or otherwise unable to communicate directly, and it would be undesirable if they could use the protocol itself to convey information.

More recently, subliminal channels have arisen as a concern in the context of cryptographic implementations of game-theoretic mechanisms. Here, informally, there is a game in which parties send their types/inputs to a trusted party which then computes an outcome/result. One might naturally want to replace the trusted party with a cryptographic protocol executed by the parties [15, 19, 6, 32, 33, 34, 28, 1, 2, 26, 24]. Using protocols for secure multi-party computation (e.g., [21]) preserves Nash equilibria; however, such protocols do *not* suffice for implementing general equilibria precisely because they have subliminal channels and thus enable collusion in the real world even if such collusion is impossible in the original game. This realization has motivated substantial effort toward constructing *collusion-free* protocols that do not allow any subliminal communication [32, 33, 34, 28, 26, 4, 3, 24]. Collusion-free protocols are impossible when parties are connected by pairwise communication channels, and so researchers have turned to other communication models. It is known that collusion-free computation of arbitrary functionalities is possible if parties have access to a semi-trusted “ballot box” and can communicate publicly via (physical) envelopes [33, 28, 26, 24], or if parties are connected (via standard communication channels) to a semi-trusted entity in a “star network” topology [4, 3].

## 1.1 A New Definition: Collusion Preservation

The works of Izmalkov et al. [33, 28, 26, 24] and Alwen et al. [4, 3] give incomparable definitions of collusion freeness, each tailored (to some extent) to the specific communication models under consideration. We revisit these definitions, and propose a stronger notion called *collusion preservation*.<sup>1</sup> Our aim here is to provide a *clean, general-purpose* definition that better handles *composition*, both when collusion-preserving protocols are run as sub-routines within some larger protocol, as well as when collusion-preserving protocols are run concurrently with arbitrary other protocols (whether collusion-preserving or not). In what follows, we give an overview of our definition and expound further on the above points.

**Overview of our definition.** We follow the definitional paradigm used by Alwen et al. [4, 3], and review their model here. In the real-world execution of a protocol, different adversaries can corrupt different parties. Importantly, these adversaries cannot communicate directly; instead, all parties are connected in a “star network” topology with a semi-trusted mediator. (This is in contrast to usual cryptographic definitions, which assume a “monolithic” adversary who controls all corrupted parties and can coordinate their actions.) Two notions of security are then defined, depending on whether or not the mediator is honest:

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<sup>1</sup>Collusion-free protocols ensure that if parties can communicate *nothing* without the protocol, then they can communicate nothing with the protocol. Collusion-preserving protocols provide a stronger guarantee: that *whatever* parties can communicate without the protocol, they can communicate no more with the protocol. See further discussion below.

CONDITIONAL COLLUSION FREENESS: When the mediator is honest, collusion freeness is required.

This is formalized by considering an ideal world where there are similarly adversaries who cannot communicate directly, but can only send inputs to (and receive outputs from) an ideal functionality. A protocol is collusion-free if for any PPT real-world adversaries interacting with the (honest) mediator, there are PPT ideal-world adversaries such that the real and ideal worlds are indistinguishable. Indistinguishability here is formulated in a “stand-alone” manner [20].

FALLBACK SECURITY: When the mediator is dishonest, we clearly cannot hope for collusion freeness any more. Nevertheless a strong and meaningful notion of security can be achieved. Namely, adversaries are now allowed to communicate arbitrarily (in both the real and ideal worlds), and the protocol is required to satisfy the standard notion of stand-alone security [20].

We strengthen and extend the definition of collusion freeness in several ways. First, rather than considering a specific “star network” topology, or the specific physical assumptions of [33, 28, 26, 24], we consider a general *resource* to which the parties have access in the real world. (This resource is the only means of “legal” communication in the real world, though as we will see in a moment there may be other “illicit” means of communication available.) In addition to the inherent advantages of a more general definition, formulating a generic definition allows us to characterize the minimal properties that resources need in order to achieve collusion-preserving computation.

An additional difference is that we formulate our definitions in a universally composable (UC) [8] fashion, where there is an environment controlling the entire execution. (Actually, we use the *generalized* UC framework [10] as our starting point.) This has significant ramifications, since the environment *itself* can now act as a communication channel for the adversaries. If the environment chooses to allow no communication between the adversaries, then our definitions essentially “default” to the previous setting of collusion freeness. Crucially, however, if the environment allows the adversaries to communicate  $c$  bits of information “for free”, then a collusion-preserving protocol ensures that the adversaries cannot communicate more than  $c$  bits (on top of the communication allowed by the ideal functionality) by running the protocol in the presence of the stated resource. (We show below a simple counter-example demonstrating that collusion freeness does not imply collusion preservation.) Moreover, we obtain composition “for free” due to the power of the UC framework. In this way we improve upon the results of [4, 3], which do not claim nor realize any form of composition, as well as the results of [33, 28, 26, 24] which obtain only a limited sort of composition; see below.

**Collusion preservation is stronger than collusion freeness.** We give a simple counter-example showing that collusion preservation is stronger than collusion freeness. Consider a protocol  $\pi$  that is collusion-free in the communication model of [4, 3] where, in short, there is a central mediator to whom all parties are connected (and no other channels are available). We obtain a new protocol  $\pi'$ , identical to  $\pi$  except for the following two modifications to the mediator’s behavior (where,  $\lambda$  is the security parameter):

1. The mediator takes a special message  $m \in \{0, 1\}^{2\lambda}$  from  $P_1$ . In response, the mediator chooses a random  $r \in \{0, 1\}^\lambda$ , sends it to  $P_1$ , and stores  $(r, m)$ .
2. The mediator takes a special message  $m' \in \{0, 1\}^\lambda$  from  $P_2$ . If the mediator has a stored tuple of the form  $(r', m)$ , it sends  $m$  to  $P_2$  (and otherwise simply ignore  $r$ ).

It is not hard to see that  $\pi'$  remains collusion-free: intuitively, this is because  $P_2$  can guess  $r' = r$  with only negligible probability.<sup>2</sup> However,  $\pi'$  is *not* collusion preserving. Specifically, if  $P_1$  and  $P_2$  have access to a  $\lambda$ -bit channel then they can use  $\pi'$  to communicate  $2\lambda$  bits!

One can interpret this counter-example in several ways. One could imagine that  $\pi'$  is run in a setting in which  $P_1$  and  $P_2$  have access to a physical channel that only allows communication of  $\lambda$  bits. Alternately, the parties might be running  $\pi'$  while they are concurrently running some *other* protocol that is not collusion-free and enables the parties to (subliminally) communicate  $\lambda$  bits. Either way, the implication is the same: a collusion-free protocol may potentially allow additional communication once parties have the ability to communicate at all.

**Protocol composition.** A critical feature of the protocols of Izmalkov et al. [33, 28, 26, 24] is that they are only collusion-free when *at least one party running the protocol is honest*. The underlying reason is that in their communication model parties have *the ability* to communicate arbitrary information; the guarantee provided by their protocols is that any such communication will be detected. Collusion freeness thus requires an honest party to perform the detection.

This limitation may not appear to be a problem, since one typically does not care to provide any guarantees once all parties are malicious. It becomes a problem, however, when collusion-free protocols are used as sub-routines within some larger protocol. Consider, for example, a collusion-free protocol  $\Pi$  for three parties  $P_1, P_2, P_3$  in which each pair of parties runs some collusion-free sub-protocol  $\pi$  between themselves. If  $P_1$  and  $P_2$  are malicious, then  $\pi$  may provide no guarantees which means that they may now be able to communicate an unlimited amount of information; this could clearly be problematic with regard to the “outer protocol”  $\Pi$ . (Izmalkov et al. implicitly avoid this issue by having all parties take part in any sub-protocols that are run. While this solves the issue, it is not an efficient approach.)

**General feasibility with GUC fallback.** Complementing and motivating our definitional work, we provide a completeness result for strong realization of a large class of functionalities. More concretely, for any functionality in this class we provide a protocol compiler and a particular resource which satisfies a universally composable version of the security definition from [3]: the compiled protocol provides Collusion Preserving (CP) security when executed with this particular resource, and, as a strong fallback, when executed with an arbitrary resource, it achieves GUC-type security, i.e., emulation by “monolithic” simulators.

**Implications for computational game theory.** The CP framework is defined in terms of computationally bounded parties. As such the implications for game theory concern games with similar limitations as in the field of algorithmic game theory [37]. We consider these (extensive form) computational games to be equivalent in some sense if they contain the same set of computational Nash Equilibria (cNE) which also implies that stronger kinds equilibria are also preserved.

Translating the new security notion and our general feasibility result to the setting of computational mediated games we show how to replace a mechanism with a protocol running over a “less trusted” mechanism such that the resulting game is equivalent to the original. By less trusted we mean that the mechanism need no longer be trusted to enforce privacy, nor to compute the game round functions correctly. (However it is still trusted to enforce both fairness and the isolation of players.)

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<sup>2</sup>In particular the simulators for  $\pi'$  can behave just as for  $\pi$  with the only modification that the simulator for  $P_1$  responds with a random message  $r$  when it receives the special message  $m$  from it’s adversary.

In comparison to results of [25, 27, 24] which provide information theoretic-equivalence between games using an unconventional model of computation, the results in this paper provide only computational equivalence but use a standard computational model. However their notion of composition is weaker in two ways. Conceptually it is not scalable but more concretely it seems to allow for only rather limited notion of concurrency. In particular protocols that implement a mechanism must be run atomically with respect to actions in any concurrent games. In the notion obtained in this paper is fully UC composable in the more traditional sense. On the other hand while our protocols prevent signaling via aborts as in [25], they do not provide the full robustness to aborts of [27, 24]. Finally the amount of randomness in the public view of our protocols is limited to a single pre-computation round which can be run before types are distributed. From that point on there is no further “randomness pollution”. This is similar to [31], better than [25] (where even executions of a protocol produce randomness pollution) but weaker than [27, 24] which do not produce any randomness pollution at all.

**Relations to Abstract Cryptography.** Maurer and Renner introduced the Abstract Cryptography (AC) framework [35]. In Appendix B we describe how our model can be specified using the language of the Abstract Cryptography framework. Besides elucidating parallels between these two frameworks, to the best of our knowledge the following discussion represents the first concrete instantiation of AC.

**Outline of this paper.** In Section 2 we define a Universally Composable version of Collusion Freeness, called Collusion Preservation; we prove its composability and show how it relates to existing universally composable security notions. In Section 3 we characterize three minimal assumptions any resource must have if it is to be used for realizing some important classes of functionalities; our characterization rules out the use of standard communications models to implement almost any interesting functionality. Next, in Section 5, we prove our general feasibility result by providing a protocol compiler and a particular resource with which a large class of functionalities can be CP implemented in a strong way that guarantees a GUC-type fallback. In Section 6 we show how to use the protocol compiler to significantly reduce the trust in a mechanism for games with computationally bounded players.

## 2 Collusion-Preserving Computation

In this section we define our framework for investigating universally composable collusion freeness, namely *collusion-preserving* computation. On the highest level the idea is to combine the strong composability properties of the GUC framework of [11] with the model of split simulators along the lines of [3].

### 2.1 Preliminaries and Notation

We denote by  $[n]$  the set  $\{1, \dots, n\}$  (by convention  $[0] = \emptyset$ ) and for a set  $\mathcal{I} \subseteq [n]$  we write  $\bar{\mathcal{I}}$  to denote the set  $[n] \setminus \mathcal{I}$ . Similarly, for element  $i \in [n]$  we write  $\bar{i}$  to denote the set  $[n] \setminus \{i\}$ . Using this notation we denote by  $\mathbf{A}_{\mathcal{I}}$  a set of ITMs  $\{\mathbf{A}_i\}_{i \in \mathcal{I}}$ . For input tuple  $x_{\mathcal{I}} = \{x_i\}_{i \in \mathcal{I}}$  we write  $\mathbf{A}_{\mathcal{I}}(x_{\mathcal{I}})$  to denote that for all  $i \in \mathcal{I}$  the ITM  $\mathbf{A}_i$  is run with input  $x_i$  (and a fresh uniform independent random tape).

Additionally we will make heavy use of elements from the GUC framework such GUC protocols and GUC setup functionalities. For reasons of space we assume passing familiarity with the model

used their in. For a more detailed description of the main features of GUC used to describe our framework we refer to Appendix A.

## 2.2 An Intuitive Description.

Starting from the GUC model we make the following modifications:

**SPLIT ADVERSARIES/SIMULATORS:** Instead of a monolithic adversary/simulator we consider a set of  $n$  (independent) PPT adversaries  $A_{[n]} = \{A_i : i \in [n]\}$ , where  $A_i$  correspond to the adversary associated with the player  $i$  (and can corrupt at most this party). Moreover, we ask that for each  $A_i \in A_{[n]}$  there exists an (independent) simulator  $Sim_i$ .

**CORRUPTED-SET INDEPENDENCE:** We also require that the simulators do not depend on each other. In other words the code of simulator  $Sim_i$  is the same for any set of adversaries  $A_{[n]}$  and  $B_{[n]}$  as long as  $A_i = B_i$ .

**Modeling Split Adversaries.** To incorporate the notion of a split adversary in the GUC model, we make the following modifications to the model of execution: Let  $\mathcal{P} = [n]$  be the player set; instead of a single adversary, we introduce to the model  $n$  independent adversaries  $A_1, \dots, A_n$ , such that for each  $P_i \in \mathcal{P}$ ,  $A_i$  might corrupt at most party  $p_i$ .<sup>3</sup> For this purpose, each adversary is associated with a unique adversary-ID, which includes the party-ID of the corresponding party. Each of the adversaries  $A_i$  has a dedicated interface to communicate with  $\mathcal{Z}$ .<sup>4</sup> Note that, unlike the standard (G)UC models, the adversaries do not serve as an underlying insecure network, as they do not share communication tapes with the honest parties or with each other. In order to make statements about computation of non-trivially computable, i.e., non-locally computable, functionalities one needs to consider hybrid worlds, where the hybrid serves as the communication resource for the parties and/or as the “co-ordination” mechanism for the adversaries. In the following we give a generic specification of the mode of operation of functionalities in our split adversaries setting and sketch the model of execution of protocol in the corresponding hybrid-model. The difference between the multi-adversary hybrid world execution and the standard (G)UC execution is graphically represented in Figure 1.

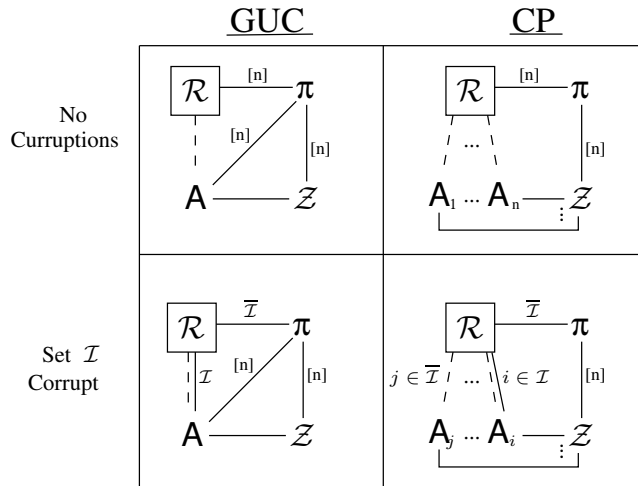


Figure 1: UC corruptions compared to CP corruptions.(Setup functionalities are left implicit.)

**Resources, Shared Functionalities, and Exclusive Protocols.** The main *difference* between a CP functionality  $\mathcal{F}$  and GUC one is that besides the  $n$  interfaces to the (honest) parties it also has interfaces to *each* of the  $n$  adversaries  $A_1, \dots, A_n$ . In other words rather than  $n$  interfaces a CP functionality has  $2n$  interfaces.

<sup>3</sup>The corruption mechanism is similar to the (G)UC setting, i.e., the environment  $\mathcal{Z}$  requests  $A_i$  corrupt  $P_i$ .

<sup>4</sup>Technically, as in the UC setting, each such interface corresponds to  $A_i$  and  $\mathcal{Z}$  sharing a communication tape.

Moreover, similar to the GUC framework (but in contrast to plain UC) we distinguish between two types of functionalities: *resources* which we denote with capital calligraphic font as in “ $\mathcal{R}$ ” and *shared functionalities* which we denote with an additional over-line as in “ $\overline{\mathcal{G}}$ ”. Formally a resource  $\mathcal{R}$  maintains state only with respect to a single instance of a protocol, while a *shared* functionality  $\overline{\mathcal{G}}$  can maintain state across protocol instances.<sup>5</sup> For example concurrent executions can maintain shared state via say a global CRS (like the ACRS of the GUC framework) or via a global PKI (such as the KRK setup for GUC) as long as these are modeled as shared functionalities. However, although concurrent instances of a protocol  $\pi$  may use the same resource  $\mathcal{R}$ , the behavior of  $\mathcal{R}$  in one execution of  $\pi$  must be independent of all other executions of  $\pi$  (and more generally of all other concurrent protocols instantiated by the environment). For clarity, in the remainder of this work we will usually refer to shared functionalities simply as *setup* and protocols which only share state across executions through some setup  $\overline{\mathcal{G}}$  as  *$\overline{\mathcal{G}}$ -subroutine respecting*. As observed already in previous works, e.g., see [11], in practice most protocols are in fact subroutine respecting with respect to the shared functionality which they use as setup.<sup>6</sup>

**The  $\mathcal{R}$ -Hybrid World.** A CP execution in the  $\mathcal{R}$ -hybrid world is defined via a straightforward generalization to the analogous GUC execution. In particular when the environment  $\mathcal{Z}$  requests corruption of a player  $P_i$  the adversary  $A_i$  is given control of  $P_i$ 's interface to  $\mathcal{R}$  (c.f. Figure 2). On a technical level the execution of an  $\mathcal{R}$ -hybrid protocol is almost identical to an execution in the GUC framework. In particular, environments can instead invoke an arbitrary number of ITI of any kind; even those running other protocols and sharing state via some setup. We denote the output of the environment  $\mathcal{Z}$  when witnessing an execution of protocol  $\pi := \pi_{[n]}$  attacked by adversaries  $A := A_{[n]}$  in the  $\mathcal{R}$ -hybrid model as  $\text{CP-EXEC}_{\pi, A, \mathcal{Z}}^{\mathcal{R}}$ . Finally, we say a protocol  $\pi$  is  *$\mathcal{R}$ -exclusive* if it makes use of no other resources (shared or otherwise) than  $\mathcal{R}$ .

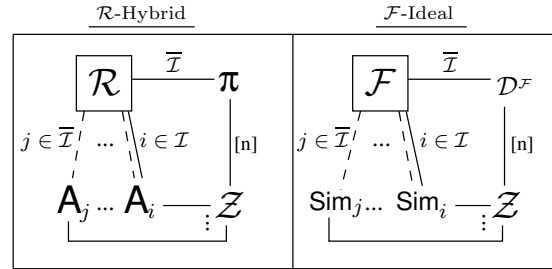


Figure 2:  $\mathcal{R}$ -hybrid vs.  $\mathcal{F}$ -ideal CP executions where playerset  $\mathcal{I} \subseteq [n]$  has been corrupted. (Setup functionalities are left implicit.)

### On Bounding the Number of Calls to Resources.

A primary difference between how executions in a  $\mathcal{R}$ -hybrid world are modeled in the GUC and CP frameworks is that in the CP case parties can communicate with at most a *single* instance of  $\mathcal{R}$ . This is in contrast to all other UC like models where say an OT-hybrid world is understood to mean that parties can make as many calls as they wish to the OT functionality instantiating a new copy for each new OT transaction they wish to perform.

At first glance this may seem like a rather minor modeling issue since anyway environments are not always aware of the presence of ideal functionalities (otherwise distinguishing hybrid world would be trivial). However we argue that, in contrast to a setting with a monolithic adversary where no such restriction is made, for a composable notion with split adversaries fixing the number of instances of functionalities (i.e. resources) available to adversaries is in fact *crucial* for capturing the desired intuition of collusion freeness.

For example a primary motivation of this work is to provide a way for reducing trust in the

<sup>5</sup>Technically this is modeled by restricting an instance of a resource to only accept inputs from ITI with a fixed session ID value while a shared functionality is can accept inputs from an ITI with *any* session ID.

<sup>6</sup>This notion is completely analogous to that of “subroutine respecting” as defined for the (G)UC frameworks.



mediators used in mechanism design by providing protocols which can be used to replace the interaction with the mediator. However if we do not restrict the number of instances of the mechanism with which parties can interact then there is no meaningful way to capture a game which calls for only a single instance.

From a cryptographic point of view, suppose we prove that some protocol  $\pi$  “realizes” a functionality  $\mathcal{F}$  *without* fixing the number of instances of  $\mathcal{F}$  simulators can interact with. Intuitively, this would mean  $\pi$  allows as much collusion as can be obtained by an *unlimited* number of calls to  $\mathcal{F}$ . If  $\mathcal{F}$  is a one-bit bi-directional channel for example the meaning of the statement changes completely if simulators use only a single instance of  $\mathcal{F}$  to simulate versus when they can use unlimited calls to  $\mathcal{F}$ .

We contrast this with the case of a monolithic adversary where no such issue arises (and indeed, the definition of (G)UC seem to allow for such simulations). The underlying reason is that even in the ideal world there is only a single simulator. Thus any correlation obtained by the simulators in the previous example for say player  $i$  and player  $j$  from the first execution of  $\pi$  is now irrelevant as the same correlation can be simulated trivially internally by the monolithic simulator controlling both parties. Thus no intuition is lost by not fixing how many calls to the ideal functionality are made for simulation purposes.

**Definition 2.1** (Collusion-Preserving Computation). *Let  $\bar{\mathcal{G}}$  be a setup,  $\mathcal{R}$  and  $\mathcal{F}$  be  $n$ -party resources,  $\pi$  be a  $\{\bar{\mathcal{G}}, \mathcal{R}\}$ -exclusive protocol and  $\phi$  be a  $\{\bar{\mathcal{G}}, \mathcal{F}\}$ -exclusive protocol (both  $n$ -party protocols). Then we say that  $\pi$  collusion-preservingly (CP) emulates  $\phi$  in the  $\{\bar{\mathcal{G}}, \mathcal{R}\}$ -hybrid world, if there exists a collection of efficiently computable transformations  $\text{Sim} = \text{Sim}_{[n]}$  mapping ITMs to ITMs such that for every set of adversaries  $\mathbf{A} = \mathbf{A}_{[n]}$ , and every PPT environment  $\mathcal{Z}$  the following holds:*

$$\text{CP-EXEC}_{\pi, \mathbf{A}, \mathcal{Z}}^{\bar{\mathcal{G}}, \mathcal{R}} \approx \text{CP-EXEC}_{\phi, \text{Sim}(\mathbf{A}), \mathcal{Z}}^{\bar{\mathcal{G}}, \mathcal{F}}$$

**Realization, Reductions and the “ $\sqsubseteq$ ” Notation.** As in the (G)UC frameworks, we distinguish between the more general notion of “emulation” and the special case of “realization”. For an (implicit) functionality  $\mathcal{F}$  we denote by  $\mathcal{D}_i^{\mathcal{F}}$  the  $i^{\text{th}}$  *dummy  $\mathcal{F}$ -hybrid protocol* which simply acts as a transparent conduit between the  $i^{\text{th}}$  honest and adversarial interfaces of  $\mathcal{F}$  and  $\mathcal{Z}$ . In particular  $\mathcal{D}_i^{\mathcal{F}}$  forwards all messages it receives from  $\mathcal{Z}$  to the functionality  $\mathcal{F}$  (where the choice of adversarial or honest interface is specified by  $\mathcal{Z}$ ) and vice-versa. If for functionality  $\mathcal{F}$ , an  $\mathcal{R}$ -hybrid protocol  $\pi$  CP-emulates  $\mathcal{D}^{\mathcal{F}}$  then we say that  $\pi$  *realizes  $\mathcal{F}$  (in the  $\mathcal{R}$ -hybrid world)*. In symbols we denote this by  $\mathcal{F} \sqsubseteq_{\pi}^{\text{CP}} \mathcal{R}$ , which can intuitively be read as “ $\mathcal{F}$  CP-reduces to  $\mathcal{R}$  via protocol  $\pi$ ”.<sup>7</sup> By omitting  $\pi$  in this notation we denote simply the existence of some protocol for which the relation holds. We also use “ $\sqsubseteq^{\text{GUC}}$ ” to denote the analogous relation but for *GUC*-realization.

By convention for any functionality  $\mathcal{R}$  we consider a pair of protocols equivalent  $\pi = \phi$  in the  $\mathcal{R}$ -hybrid world if for all functionalities  $\mathcal{F}$  we have:

$$\mathcal{F} \sqsubseteq_{\pi}^{\text{CP}} \mathcal{R} \iff \mathcal{F} \sqsubseteq_{\phi}^{\text{CP}} \mathcal{R}$$

In particular let  $\mathcal{D}^{\mathcal{R}}$  be the dummy protocol for  $\mathcal{R}$ . Then by convention, for any protocol  $\pi$  we have  $\pi^{\mathcal{D}^{\mathcal{R}}} = \pi$  in the  $\mathcal{R}$ -hybrid world where  $\pi^{\mathcal{D}^{\mathcal{R}}}$  is the protocol behaving as  $\pi$  but with calls to  $\mathcal{R}$  being forwarded through  $\mathcal{D}^{\mathcal{R}}$  as if they came from  $\mathcal{Z}$ .

To simplify notation and maintain consistency with previous UC-type works, whenever an explicit protocol for the honest players is missing in the CP-EXEC notation then it is implicitly assumed that they are running  $\mathcal{D}^{\mathcal{F}}$ . For example we might write  $\text{CP-EXEC}_{\mathbf{A}_{[n]}, \mathcal{Z}}^{\mathcal{F}}$  when the honest

<sup>7</sup>Alternatively it can also be read as “Protocol  $\pi$  CP-realizes  $\mathcal{F}$  in the  $\mathcal{R}$  hybrid model.”

players are running  $\mathcal{D}_{[n]}^{\mathcal{F}}$ . For clarity, we include a graphical representation of the multi-adversary hybrid-world/ideal-world definitions in Figure 2.

## 2.3 Composition Theorem and Other Tools

We formalize a strong (universally) composable property of CP security and, along the way, provide a useful tool for proving CP security of protocols.

### 2.3.1 Simplified CP

Following the approach of [11] we make two simplifications to the definition of CP security obtaining what we call *Simplified CP (SCP)*.<sup>8</sup> SCP has two desirable properties:

1. It significantly easier to prove a protocol SCP secure then CP secure.
2. Yet SCP security is equivalent to CP security under certain reasonable conditions.

Combining these two property also results in a simpler proof of the main UC-type composition theorem then if we try and prove it directly for CP security.

To simplify the task of proving protocols SCP secure we restrict the class of environments being considered (much like "Externalized UC" of [11]). Let  $\bar{\mathcal{G}}$  be a setup,  $\mathcal{R}$  be a resource and  $\pi$  be an  $(\bar{\mathcal{G}}, \mathcal{R})$ -exclusive protocol.<sup>9</sup> Then for SCP security we quantify only over *restricted environments* which are PPT environments which do not invoke any other ITMs besides a single instance of protocol  $\pi$  as well as one instance of the dummy protocol  $\mathcal{D}^{\bar{\mathcal{G}}}$ . The second modification we make to CP is that we quantify only over dummy adversaries. That is for all  $i \in [n]$  we consider only the adversary  $A_i$  which acts as a conduit between  $\mathcal{Z}$  and the resource  $\mathcal{R}$ .

Proving SCP security is significantly easier than proving CP security, because only a single kind of simulator need be considered (namely for the dummy adversaries). Moreover when verifying the correctness of these simulators it is easier (tractable) to reason about the entire view of an environment if it's view consists of only a single execution of the protocol and the interaction with the shared functionality.

**Definition 2.2** (Simplified Collusion-Preserving Computation). *Let  $\bar{\mathcal{G}}$  be a setup,  $\mathcal{R}$  and  $\mathcal{F}$  be  $n$ -party resources,  $\pi$  be a  $\{\bar{\mathcal{G}}, \mathcal{R}\}$ -exclusive protocols and  $\phi$  be a  $\{\bar{\mathcal{G}}, \mathcal{F}\}$ -exclusive protocols (both  $n$ -party protocols). Let  $\mathbf{B} = \mathbf{B}_{[n]}^{\pi}$  be the set of  $n$  dummy adversaries interacting with  $\pi$ . Then we say that  $\pi$  Simplified CP (SCP) emulates  $\phi$  in the  $\{\bar{\mathcal{G}}, \mathcal{R}\}$ -hybrid world, if there exists a collection of efficiently computable transformations  $\text{Sim} = \text{Sim}_{[n]}$  mapping ITMs to ITMs such that for every restricted environment  $\mathcal{Z}$  it holds that:*

$$\text{CP-EXEC}_{\pi, \mathbf{B}, \mathcal{Z}}^{\bar{\mathcal{G}}, \mathcal{R}} \approx \text{CP-EXEC}_{\phi, \text{Sim}(\mathbf{B}), \mathcal{Z}}^{\bar{\mathcal{G}}, \mathcal{F}}.$$

**Equivalence Theorem.** We first show that although SCP security may at first glance seem weaker then CP security, in fact they are equivalent. As in the GUC framework, results relating SCP to CP are conditioned on protocols being subroutine respecting. Intuitively this is because only if a protocol is  $\bar{\mathcal{G}}$ -subroutine respecting can it be guaranteed that giving the environment access to  $\bar{\mathcal{G}}$  is sufficient to faithfully internally emulate multiple concurrent executions of the protocol.

**Theorem 2.3** (Equivalence). *Let  $\bar{\mathcal{G}}$  be a setup,  $\mathcal{R}$  and  $\mathcal{F}$  be resources,  $\pi$  be a  $(\bar{\mathcal{G}}, \mathcal{R})$ -exclusive protocol and  $\phi$  be a  $(\bar{\mathcal{G}}, \mathcal{F})$ -exclusive protocol. Then  $\pi$  CP emulates  $\phi$  if and only if  $\pi$  SCP emulates  $\phi$ .*

<sup>8</sup>This can be thought of as being analogous to Externalized-UC combined with the Dummy Lemma.

<sup>9</sup>Note that this implies that  $\pi$  is also  $\bar{\mathcal{G}}$ -subroutine respecting protocol.

*Proof.* As the set of environments and adversaries for SCP are a subset of those for CP so the forwards direction (“ $\implies$ ”) is trivially true.

The backwards direction (“ $\impliedby$ ”) requires more work. It combines in a straight forward manner the ideas of the proof of the so called “dummy lemma” of [9] with those from the proof of Theorem 2.1 [11]<sup>10</sup>. Indeed the original proofs translate almost one-to-one and all that is needed is to verify that splitting the adversaries does not cause a break down in the logical arguments.

For completeness we sketch the entire proof highlighting the changes required to accommodate split adversaries. We first argue that quantifying over fewer environments does not change the quality of the security notion. Then we argue that considering only dummy adversaries also has no effect on the quality of the security notion which concludes the proof.

We show that for any  $\mathcal{Z}$  in the CP framework there exists a restricted environment  $\tilde{\mathcal{Z}}$  with at most polynomial loss in advantage at distinguishing executions of  $\phi$  from  $\pi$ . This breaks down into two arguments:

1. Instantiating polynomially many arbitrary concurrent protocols communicating with  $\bar{\mathcal{G}}$  does not help  $\mathcal{Z}$  as  $\tilde{\mathcal{Z}}$  has a direct link to  $\bar{\mathcal{G}}$  (via the dummy protocol for  $\bar{\mathcal{G}}$ ) and so can perfectly emulate all concurrent protocols internally.
2. Interacting with polynomially many instances of  $\pi$  does not help  $\mathcal{Z}$  significantly via a standard hybrid argument. The core of the hybrid argument is that there must be an  $l^{\text{th}}$  concurrent execution of  $\pi$  that helps  $\mathcal{Z}$  distinguish significantly and so  $\tilde{\mathcal{Z}}$  can simulate the previous  $l-1$  execution and use the  $l^{\text{th}}$  as the real one. Therefore there is at most a polynomial loss in the advantage of  $\tilde{\mathcal{Z}}$  when compared to the advantage of  $\mathcal{Z}$ .

Roughly speaking, we have shown that a protocol looks the same to all efficient environments only if it looks the same to all restricted environments. It remains to show that for any distinguishing restricted environment interacting with *arbitrary* adversaries there exists a distinguishing restricted environment interacting only with dummy adversaries. The proof remains essentially unchanged from that of the dummy lemma in [9] so we only briefly mention how to extend it to the SCP setting. In particular it relies on the idea that the simulator for dummy adversaries can be used as “on-line translators” turning views of  $\phi$  into views of  $\pi$ .

The original lemma (Claim 10 in [9]) concerns a single dummy adversary  $B^\pi$  attacking  $\pi$  (potentially controlling multiple players  $\mathcal{I} \in [n]$  rather than a set of individual dummies each controlling only a single player). By assumption there exists a simulator  $\tilde{\text{Sim}}(B^\pi)$  attacking execution(s) of  $\phi$  that is able to create an indistinguishable view of execution(s) of  $\pi$  for all players it controls which it feeds to the environment (just as  $B^\pi$  would when attack  $\pi$ ). The proof uses this fact to construct, for any adversary  $A$  attacking  $\pi$ , a simulator  $\text{Sim}(A)$  attacking  $\phi$  with similar capabilities as  $\tilde{\text{Sim}}(B^\pi)$ . More precisely, for an arbitrary adversary  $A$  the proof describes a simulator  $\text{Sim}(A)$  which internally emulates  $A$  and let’s it interact with an internal emulation of  $\tilde{\text{Sim}}$  which is used to translate the view of  $\phi$  into a matching view of  $\pi$  for  $A$  to attack. To see that  $\text{Sim}(A)$  is a good simulator it is observed that an environment  $\mathcal{Z}$  which can distinguish between  $\text{EXEC}_{\pi_{\mathcal{I}}, A_{\mathcal{I}}, \mathcal{Z}}^{\bar{\mathcal{G}}}$  and  $\text{EXEC}_{\phi_{\mathcal{I}}, \tilde{\text{Sim}}_{\mathcal{I}}(A), \mathcal{Z}}^{\bar{\mathcal{G}}}$  can be used to break the correctness of  $\tilde{\text{Sim}}(B^\pi)$ . All that is needed is for an environment  $\mathcal{Z}'$  to run  $\mathcal{Z}$  and  $A$  internally and let them interact directly with  $\tilde{\text{Sim}}(B^\pi)$  or  $B^\pi$ .

The same argument carries through to our setting. In the SCP framework, given a set of  $n$  simulators which work for any subset of dummy adversaries, the same construction of simulators for sets of arbitrarily corrupt players will work. Let  $A_{[n]}$  be a set of arbitrary adversaries attacking  $\pi$ . The goal is to describe an (efficient) transformation  $\text{Sim}_i$  mapping  $A_i$  to a corresponding simulator

<sup>10</sup>Theorem 2.1 in [11] states that GUC and EUC are equivalent.

$\text{Sim}_i(\mathbf{A}_i)$  for all  $i \in [n]$ . By assumption, for dummy adversaries  $\mathbf{B}_{[n]}^\pi$  interacting with protocol  $\pi$  SCP security already provides for a set of transformations  $\tilde{\text{Sim}}_{[n]}$  such that the simulators  $\tilde{\text{Sim}}_{[n]}(\mathbf{B}_{[n]}^\pi)$  interact with an execution of  $\phi$  and can simulate all dummy adversaries' views of  $\pi$  faithfully to any (restricted) environment. For each  $i \in [n]$  we construct  $\text{Sim}_i(\mathbf{B}_i^\pi)$  from  $\tilde{\text{Sim}}_i(\mathbf{B}_i^\pi)$  just as in the original proof.

It remains to show that these are good constructions. Formally we must show that no environment  $\mathcal{Z}$  has significantly different output for  $\text{CP-EXEC}_{\pi, \mathbf{A}_{[n]}, \mathcal{Z}}^{\bar{\mathcal{G}}, \mathcal{R}}$  and for  $\text{CP-EXEC}_{\phi, \text{Sim}_{[n]}(\mathbf{B}_{[n]}^\pi), \mathcal{Z}}^{\bar{\mathcal{G}}, \mathcal{F}}$ . Suppose for sake of contradiction that  $\mathcal{Z}$  can distinguish between these two executions. In this case we can build restricted environment  $\mathcal{Z}'$  which breaks the SCP security of  $\pi$  by absorbing  $\mathbf{A}_{[n]}$  and  $\mathcal{Z}$  just as in the original proof. Each  $\mathbf{A}_i$  interacts directly with the  $i^{\text{th}}$  adversary (be it  $\mathbf{B}_i^\pi$  or  $\tilde{\text{Sim}}_i(\mathbf{B}_i^\pi)$ ). Then by outputting the same bit as  $\mathcal{Z}$  environment  $\mathcal{Z}'$  has an identical advantage at breaking the SCP security of  $\pi$  as  $\mathcal{Z}$  has at breaking the simulation of  $\text{Sim}_{[n]}(\mathbf{B}_{[n]}^\pi)$ . The fact that there are  $n$  different absorbed  $\mathbf{A}_i$  each interacting with a single external adversary (rather than one  $\mathbf{A}$  interacting with all corrupt players) has no effect on the strategy of  $\mathcal{Z}'$  as it can directly communicate with all external adversaries and so does not rely on their implicit coordination via a monolithic modeling.  $\square$

### 2.3.2 A Composition Theorem

As a main motivation for the CP model we put forth the goal of providing a formal and rigorous notion of composability for collusion-free security. We capture this in the following central theorem.

**Theorem 2.4** (Composition). *Let  $\mathcal{R}$  be an arbitrary resource and  $\bar{\mathcal{G}}$  be a global setup (i.e. shared) functionality. Let  $\rho, \pi$  and  $\phi$  be  $n$ -party protocols in the  $\{\bar{\mathcal{G}}, \mathcal{R}\}$ -hybrid world such that  $\pi$  and  $\phi$  are  $\bar{\mathcal{G}}$ -subroutine respecting. If  $\pi$  SCP-emulates  $\phi$  and  $\rho$  uses  $\phi$  as a subroutine then  $\rho^{\pi/\phi}$  CP-emulates  $\rho$  in  $\{\bar{\mathcal{G}}, \mathcal{R}\}$ -hybrid world.*

*Proof.* The ideas behind the proof of GUC composition carry over directly to the setting with split simulators but for completeness we give a full proof. For further details we refer to the proof of Theorem 2.1 in [11] which can be applied almost directly to this setting.

Let  $\mathbf{B} := \mathbf{B}_{[n]}$  be the set of dummy adversaries for the  $\{\bar{\mathcal{G}}, \mathcal{R}\}$ -hybrid world. Following the same logic as in the proof of Theorem 2.3 it suffices to prove that for any efficient environment  $\mathcal{Z}$ :

$$\text{CP-EXEC}_{\rho^{\pi/\phi}, \mathbf{B}, \mathcal{Z}}^{\bar{\mathcal{G}}, \mathcal{R}} \approx \text{CP-EXEC}_{\rho, \mathbf{A}, \mathcal{Z}}^{\bar{\mathcal{G}}, \mathcal{R}}. \quad (1)$$

Since  $\pi$  and  $\phi$  are subroutine respecting it follows by Theorem 2.3 that  $\pi$  CP-emulates  $\phi$ . Thus there exist simulators  $\text{Sim} = \text{Sim}_{[n]}(\mathbf{B})$  such that

$$\text{CP-EXEC}_{\pi, \mathbf{B}, \mathcal{Z}_\pi}^{\bar{\mathcal{G}}, \mathcal{R}} \approx \text{CP-EXEC}_{\phi, \text{Sim}, \mathcal{Z}_\pi}^{\bar{\mathcal{G}}, \mathcal{R}} \quad (2)$$

for any environment  $\mathcal{Z}_\pi$ . We use  $\text{Sim}$  to construct  $\mathbf{A}$  satisfying Equation 1. Adversary  $\mathbf{A}_i$  internally runs  $\text{Sim}_i(\mathbf{B}_i)$  forwarding messages from  $\mathcal{Z}$  intended for instances of  $\pi_i$  to  $\text{Sim}_i(\mathbf{B}_i)$  instead. Moreover, any messages from  $\text{Sim}_i(\mathbf{B}_i)$  destined for either  $\mathcal{Z}$  or  $\phi_i$  are forwarded faithfully.

We note that the transformation from  $\mathbf{B}_i$  to  $\mathbf{A}_i$  is efficiently computable and moreover it depends only on the code of  $\mathbf{B}_i$ . Thus it remains to show that Equation 1 is satisfied. We do this by constructing an environment  $\mathcal{Z}_\pi$  for which Equation 2 holds if and only if Equation 1 holds. The fact that  $\pi$  SCP-emulates  $\phi$  then concludes the proof.

Intuitively  $\mathcal{Z}_\pi$  absorbs  $\mathcal{Z}$ ,  $\rho$  and all of  $\mathbf{A}$  except  $\text{Sim}_i(\mathbf{B}_i)$  internally. However for all messages  $\mathbf{A}_i$  would forward to it's internal copy of  $\text{Sim}_i(\mathbf{B}_i)$  are instead routed by  $\mathcal{Z}_\pi$  to the  $i^{\text{th}}$  (external)

adversary. This will either be  $B_i$  or  $\text{Sim}_i(B_i)$ . All responses are similarly forwarded back to  $A_i$ . Finally when  $\mathcal{Z}$  produces output  $\mathcal{Z}_\pi$  outputs the same value and terminates.

Observe that  $\text{CP-EXEC}_{\pi, B, \mathcal{Z}_\pi}^{\bar{\mathcal{G}}, \mathcal{R}} = \text{CP-EXEC}_{\rho^{\pi/\phi}, B, \mathcal{Z}}^{\bar{\mathcal{G}}, \mathcal{R}}$ . Indeed the only difference in the view of  $\mathcal{Z}$  for both executions is where any given message is sampled (but not from which distribution). Similarly  $\text{CP-EXEC}_{\phi, \text{Sim}, \mathcal{Z}_\pi}^{\bar{\mathcal{G}}, \mathcal{R}} = \text{CP-EXEC}_{\rho, A, \mathcal{Z}}^{\bar{\mathcal{G}}, \mathcal{R}}$ . Thus Equation 1 holds for  $\mathcal{Z}$  if and only if Equation 2 hold for  $\mathcal{Z}_\pi$ .  $\square$

## 2.4 Relations to Existing Security Notions.

We prove a pair of lemmas relating CP results to matching GUC results. In Lemma 2.5 we show to translate a statement about CP realization into a related statement about GUC realization. Then in Lemma 2.6 we show how to convert statements in the opposite direction. Together these results capture the intuitive claim that the CP model is at least as expressive as the GUC (and so also UC) models.

### 2.4.1 CP Implies GUC.

We first formalize the intuitive claim that CP security is at least as strong as GUC security with the lemma which states that CP realization essentially implies the GUC realization. We describe a natural mapping of CP functionalities to analogous GUC functionalities  $\mathcal{F} \mapsto [\mathcal{F}]$ . Then we show that if a protocol CP realizes  $\mathcal{F}$  in the  $\mathcal{R}$ -hybrid world, then *the same* protocol executed in the analogous GUC  $[\mathcal{R}]$ -hybrid world GUC realizes the analogous GUC functionality  $[\mathcal{F}]$ .

Recall that the only difference between (possibly shared) GUC and CP functionalities is that the former has a single adversarial interface while the latter has  $n$  such interfaces. The mapping  $[\cdot]$  is obtained as follows. For a CP functionality  $\mathcal{F}$ , let  $[\mathcal{F}]$  denote the GUC functionality which behaves as  $\mathcal{F}$  except that the adversaries interface has the following modification:

- Whenever  $\mathcal{F}$  would output a message  $\text{Msg}$  to the  $i^{\text{th}}$  adversarial interface,  $[\mathcal{F}]$  instead outputs  $(i, \text{Msg})$  on it's (single) adversarial interface.
- Whenever  $[\mathcal{F}]$  receives a message of the form  $(i, \text{Msg})$  on it's adversarial interface, it simulates the behavior of  $\mathcal{F}$  upon receiving message  $\text{Msg}$  on the  $i^{\text{th}}$  adversarial interface.

**Lemma 2.5.** *Let  $\bar{\mathcal{G}}$  be a setup and  $\mathcal{F}$  and  $\mathcal{R}$  be functionalities all in the CP model. If a protocol  $\pi$  CP-realizes  $\mathcal{F}$  in the  $\{\bar{\mathcal{G}}, \mathcal{R}\}$ -hybrid model, then  $\pi$  GUC-securely realizes  $[\mathcal{F}]$  in the  $\{[\bar{\mathcal{G}}], [\mathcal{R}]\}$ -hybrid model. In symbols:*

$$\{\bar{\mathcal{G}}, \mathcal{F}\} \sqsubseteq_{\pi}^{\text{CP}} \{\bar{\mathcal{G}}, \mathcal{R}\} \implies \{[\bar{\mathcal{G}}], [\mathcal{F}]\} \sqsubseteq_{\pi}^{\text{GUC}} \{[\bar{\mathcal{G}}], [\mathcal{R}]\}$$

### 2.4.2 Translating GUC Statements to CP Statements

As a primary building block for feasibility results we will use GUC protocols. To show how to translate GUC results into analogous statements in the CP framework we first need to formalize the CP analogue of the insecure channels functionality.

**The  $\mathcal{R}_{\text{ins}}$  Functionality.** Intuitively, the main differences between the two models are that in the CP model:

1. In the plain CP model is not equipped with insecure channels (as is the case for GUC).
2. CP adversaries are split and so cannot, a priori, coordinated their attacks arbitrarily.
3. The GUC model explicitly assumes authenticated communication.

To address the first difference we *explicitly* equip the CP world with insecure channels. When modeled appropriately, a nice side effect is that the CP adversaries can also use the insecure channels to coordinate their attacks arbitrarily taking care of the second difference.

Formally we model the insecure channel resource (depicted in Figure 3), denoted by  $\mathcal{R}_{\text{ins}}(i, j)$ , as being parameterized by integers  $i, j \in [n]$  which denote the indices of two parties called the *sender*  $P_i$  and the *receiver*  $P_j$ . Upon receiving a message  $m$  from party:

- $P_i$ : Resource  $\mathcal{R}_{\text{ins}}(i, j)$  forwards  $m$  to adversary  $A_i$ .
- $A_i$ : Resource  $\mathcal{R}_{\text{ins}}(i, j)$  forwards  $m$  to adversary  $A_j$ .
- $A_j$ : Resource  $\mathcal{R}_{\text{ins}}(i, j)$  forwards  $m$  to receiver  $P_j$ .

We denote the complete network of insecure channels by

$$\mathcal{R}_{\text{ins}} = \{\mathcal{R}_{\text{ins}}(i, j) : (i, j) \in [n]^2, i \neq j\}.$$

Resource  $\mathcal{R}_{\text{ins}}$  is used just as the implicit insecure channels are used in (G)UC.

**Lemma 2.6.** *Let  $\bar{\mathcal{G}}$  be a (CP) setup such that there exists some  $[\bar{\mathcal{G}}]$ -subroutine respecting protocol that GUC realizes authenticated channels (over insecure channels).<sup>11</sup> Further let  $\mathcal{F}$  and  $\mathcal{R}$  be CP functionalities.*

*Then if a protocol  $\pi$  GUC realizes  $[\mathcal{F}]$  in the  $[\bar{\mathcal{G}}]$ -hybrid world, then it also CP realizes  $\{\bar{\mathcal{G}}, \mathcal{R}_{\text{ins}}, \mathcal{F}\}$  in the  $\{\bar{\mathcal{G}}, \mathcal{R}_{\text{ins}}\}$ -hybrid model with shared functionality  $\bar{\mathcal{G}}$ . In symbols:*

$$\{[\bar{\mathcal{G}}], [\mathcal{F}]\} \sqsubseteq_{\pi}^{\text{GUC}} [\bar{\mathcal{G}}] \implies \{\bar{\mathcal{G}}, \mathcal{R}_{\text{ins}}, \mathcal{F}\} \sqsubseteq_{\pi}^{\text{CP}} \{\bar{\mathcal{G}}, \mathcal{R}_{\text{ins}}\}$$

*Remark 1.* Formally, in the above lemma the syntax of the protocol  $\pi$  also needs to be adapted to the CP setting on the right hand side of the implication. For an arbitrary GUC protocol  $\pi$ , denote by  $\bar{\pi}$  the CP protocol which works just like  $\pi$ , but while  $\pi_i$  sends a message  $m$  to  $\pi_j$  via the adversary (i.e., by writing  $(m, j)$  to its communication tape shared with the adversary),  $\bar{\pi}_i$  instead uses  $\mathcal{R}_{\text{ins}}(i, j)$  to send  $m$  to  $\bar{\pi}_j$ . However, for the sake of clarity we do not distinguish between  $\pi$  and  $\bar{\pi}$  as it is clear from the context which protocol is meant.<sup>12</sup>

### 3 Necessary Assumptions for CP

Having defined the CP framework and verified its composition properties, we turn to the next major goal of this work: to provide a resource with which we can (constructively) CP-realize as many functionalities as possible. Ideally we would like to obtain a CP-complete resource: namely one from which *any* reasonable functionality can be realized. Indeed, in the next section we describe just such a resource which we call the *mediator*. However we must first justify the seemingly strong assumptions we will make when defining the mediator by showing their necessity.

To this end, we demonstrate three necessary properties a given resource must have for it to be CP-complete. As corollaries to these results we rule out realizing large classes interesting functionalities using virtually all common communication resources such as fully connected networks and

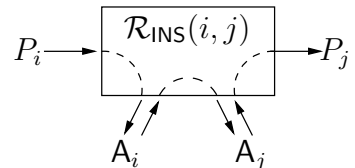


Figure 3: The Insecure Channel functionality with split adversaries.

<sup>11</sup>For example let  $[\bar{\mathcal{G}}]$  be the KRK setup of [11] for which [16] shows a protocol GUC realizing authenticated channels in the *static* corruptions setting.

<sup>12</sup>In particular for the general feasibility result in Section 5 the input to the protocol compiler is  $\bar{\pi}$  rather than  $\pi$ . This is because the fallback security (Definition 4.1) is defined in the CP setting rather than the GUC setting and so the security provided by the compiled protocol is analyzed in the presence of  $\mathcal{R}_{\text{ins}}$  rather than in the GUC model.

broadcast channels. Beyond this, due to their generality, we believe that given a target ideal functionality  $\mathcal{F}$  (such as an auction mechanism or voting functionality), these results provide significant insights into the minimal assumptions about real world communication channels which can be used to CP realize  $\mathcal{F}$ .

More precisely, in this section we prove statements of the form "if  $\mathcal{F} \sqsubseteq^{CP} \mathcal{R}$  and  $\mathcal{F}$  has property P1 then  $\mathcal{R}$  must have property P2". We provide some conditions under which  $\mathcal{R}$  must be "isolating" (Lemma 3.2), "probabilistic" (Lemma 3.4) and finally "programmable" (Lemma 3.5). Taken together we see that a CP-complete resource (such as our mediator) must simultaneously have all three properties.

The proofs extract these properties by reason about the necessary properties the behavior of  $\mathcal{R}$  must have in the setting where all players are corrupt.<sup>13</sup>

To this end we use the following reduction notion between functionalities.

**Definition 3.1.** *Let  $\mathcal{R}$  and  $\mathcal{F}$  be  $n$ -party CP resources and let  $\mathcal{D}_{[n]}^{\mathcal{F}}$  be the  $n$ -tuple of  $\mathcal{F}$ -hybrid adversaries that first corrupt their respective players and then act as the dummy adversary. We say that  $\mathcal{F}$  is contained in  $\mathcal{R}$  (written  $\mathcal{F} \sqsubseteq^C \mathcal{R}$ ) if there exists  $n$  adversaries  $\mathbf{A}_{[n]}$  (corrupting all players) such that for any efficient environment  $\mathcal{Z}$  (and protocol  $\pi$ ):*

$$\text{CP-EXEC}_{\mathcal{D}_{[n]}^{\mathcal{F}}, \mathcal{Z}}^{\mathcal{F}} \approx \text{CP-EXEC}_{\pi, \mathbf{A}, \mathcal{Z}}^{\mathcal{R}}$$

A benefit of defining the  $\sqsubseteq^C$  reduction to be so weak is that statements of the form  $\mathcal{F} \sqsubseteq^C \mathcal{R}$  are often easy to verify for many interesting resources  $\mathcal{R}$ .<sup>14</sup> This makes it a good candidate for properties of functionalities in the following results. Next we define a simple and very weak channel which we will use in place of  $\mathcal{F}$  in the properties.

**Bounded Collaborative Channels.** Let resource  $\mathcal{C}_{i,j}^c$  to be the ideal  $n$ -party functionality allowing up to  $c \in \mathbb{N}$  bits to be sent by  $P_i$  to  $P_j$  (and from adversary  $A_i$  to  $A_j$ ). Moreover  $\mathcal{C}_{i,j}^c$  requires the cooperation of all other  $n-2$  players and  $n$  adversaries in order for the message to be delivered. More precisely the message from  $P_i$  is given to both  $A_i$  and  $A_j$  who can modify it (or even halt delivery) at will. The message is only delivered to  $P_j$  once all other  $n-2$  players and adversaries have submitted a special message ok (otherwise, on any other input  $\mathcal{C}_{i,j}^c$  simply halts producing no output). We call such a channel a *bounded collaborative channel*.

We observe that statements of the type  $\mathcal{C}_{i,j}^d \sqsubseteq^C \mathcal{F}$  are often easy to verify for many interesting examples such as when  $\mathcal{F}$  is a VCG auction mechanism [41, 14, 22] (i.e. a second price auction). Even if the winners fee is kept secret from all other bidders the statement  $\mathcal{C}_{i,j}^d \sqsubseteq^C \mathcal{F}$  still holds for any value  $i, j$  and  $d$ .<sup>15</sup> On the other hand for a voting mechanism with say  $t$  candidates, private votes, public outcome and the possibility to abstain the statement holds for any value of  $i, j$  and  $d \leq \log t$ . Therefor when trying to realize these kinds relevant resources the following lemma provides an easy property to verify for candidate real world communication resources that are to be used.

<sup>13</sup>In a stand alone setting one might ask why complete corruptions are even an interesting case (for example the stand-alone collusion-free notion of [33] explicitly rules this case out). But for a composable security notion (for example with the application of modular protocol design in mind) it is vital to consider these executions since sub-protocols may be run but a set of entirely corrupt parties.

<sup>14</sup>See the following discussion on bounded collaborative channels.

<sup>15</sup>For example a set of adversaries to such that the VCG mechanism contains  $\mathcal{C}_{i,j}^d$  act as follows. Each adversary corrupts it's player (ignoring their valuations). Then  $A_j$  bids  $2^d$  on behalf of receiver  $P_j$ ,  $A_j$  bids  $m \in [0, 2^n - 1]$  on behalf of sender  $P_i$  and all other adversaries bid 0 for their players. The winner of the auction (i.e.  $P_j$ ) then outputs it's fee as the message received.

### 3.1 Isolation

Consider a statement of the type  $\mathcal{F} \sqsubseteq^{CP} \mathcal{R}$ . Intuitively this holds only if  $\mathcal{R}$  can “isolate” corrupt players as much as  $\mathcal{F}$ . Somewhat more formally suppose we wish to CP-realize functionality  $\mathcal{F}$  which allows for at most  $d$  bits of communication between players  $P_i$  and  $P_j$ . The following lemma rules out using any resource  $\mathcal{R}$  (with *arbitrary* protocol) which can be used (formally: contains) a  $c$ -bit bounded collaborative channel between these players whenever  $c > d$ . In fact, the requirements for obtaining such a channel are so weak that the lemma rules out using almost all standard communication channels as CP-complete resources as they all allow for *unbounded* amounts insecure communication between at least one pair of players.

**Lemma 3.2.** *Let  $\mathcal{R}$  and  $\mathcal{F}$  be  $n$ -player resources with  $\mathcal{C}_{i,j}^c \sqsubseteq^C \mathcal{R}$  and  $\mathcal{C}_{i,j}^d \sqsubseteq^{CP} \mathcal{F}$  then:*

$$\mathcal{F} \sqsubseteq^{CP} \mathcal{R} \implies c \leq d.$$

*Proof.* We use the transitivity of the CP relation to obtain  $\mathcal{C}_{i,j}^d \sqsubseteq^{CP} \mathcal{R}$ . Let  $A_{[n]}$  be the adversaries such that  $\mathcal{C}_{i,j}^c \sqsubseteq^C \mathcal{R}$  and suppose for the sake of contradiction that  $c > d$ . We briefly describe an environment  $\mathcal{Z}$  and  $\mathcal{R}$ -hybrid adversaries  $B_{[n]}$  for which no simulators exist fooling  $\mathcal{Z}$  in the  $\mathcal{C}_{i,j}^d$ -hybrid world. The environment samples a uniform random string  $r \leftarrow \{0,1\}^c$  and gives it to  $B_i$ . Then it waits for the output of  $r'$  of  $B_j$  and outputs 1 if and only if  $r' = r$ . Meanwhile, for each  $s \in [n]$  adversary  $B_s$  acts as  $A_s$  intuitively turning  $\mathcal{R}$  into  $\mathcal{C}_{i,j}^c$ . For  $s \notin \{i,j\}$  the adversary  $B_s$  uses ok as input to  $\mathcal{C}_{i,j}^c$  and terminates. Adversary  $B_i$  reads its input  $r$  from  $\mathcal{Z}$  and uses it as input for  $\mathcal{C}_{i,j}^c$  (on behalf of players  $P_i$ ) to be sent to  $P_j$ . Finally adversary  $B_j$  waits till it receives  $r'$  from  $\mathcal{C}_{i,j}^c$  (intended for delivery to  $P_j$ ) and outputs  $r'$  to  $\mathcal{Z}$  before terminating. It is easy to verify that  $\mathcal{Z}$  will always output 1 during such an execution.

Clearly there are no good simulators for  $B_{[n]}$  in the  $\mathcal{C}_{i,j}^d$ -hybrid world as by definition of a bounded collaborative channel there isn't enough bandwidth between the  $i^{\text{th}}$  and  $j^{\text{th}}$  honest (and adversarial) interfaces in  $\mathcal{C}_{i,j}^d$ . Thus with probability at least  $1/2$  the output of the  $j^{\text{th}}$  simulator will not equal the input to the  $i^{\text{th}}$  simulator in which case  $\mathcal{Z}$  will output 0.  $\square$

As an immediate application of Lemma 3.2 we obtain the following (informal) result ruling out most, if not all, the usual communication channels from being CP-complete. In particular, we view the following corollary provides significant justification for seaming strength of the assumptions made later on about the mediator resource. Also, the following implications for broadcast channels can be seen as extending the related impossibility result for broadcast channels from [32].

**Corollary 3.3.** *Let resource  $\mathcal{R} \sqsubseteq^{CP} \mathcal{H}$  then if  $\mathcal{H}$  is a broadcast channel then  $\mathcal{R}$  is not CP-complete. Further if  $\mathcal{H}$  is a fully connected network of insecure, authenticated or secure channels then  $\mathcal{R}$  is not CP-complete.*

**Remark On the Models of [33, 28].** When all players are corrupt their communication model contains  $\mathcal{C}_{i,j}^c$  for any value of  $c > 0$ . Thus their notion of composition requires all parties in a protocol  $\pi$  to take part in all sub-protocols of a given protocol  $\pi$  (via observation of publicly verifiable events) in order to guarantee security of  $\pi$ . On the one hand this approach requires no honest party to trust another since each can verify the computation but on the other hand this type of composition scales badly in the presence of many users. Especially in a real world setting.

### 3.2 Independent Randomness

Besides the capability of enforcing bounded isolation another requirement for realizing several interesting functionalities is that the resource being used must be a *probabilistic* ITM. This stands



in contrast with necessary assumptions on (even complete) resources in a setting with monolithic adversaries. On the other hand, in [28] for example, protocols make use of a ballot-box which contains inherent fresh randomness and the verifiable devices of [26] also have similar capabilities.

For positive integer  $c$  we denote by  $\mathcal{F}_{\text{coin}}^c$  the 2-party coin flipping functionality which takes no input and outputs  $c$  uniform random coins to  $\mathcal{P}_1$  and  $\mathcal{P}_2$ .<sup>16</sup>

**Lemma 3.4** (Randomness is Necessary). *For any integer  $c > 0$  and resource  $\mathcal{R}$  if  $\mathcal{F}_{\text{coin}}^c \sqsubseteq^{CP} \mathcal{R}$  then  $\mathcal{R}$  is a probabilistic ITM.*

The proof relies on the intuition that if  $\mathcal{R}$  is deterministic and all players are corrupt, the adversaries can completely determine the behavior of  $\mathcal{R}$  which they can use to transmit at least a bit between each other.

*Proof.* Suppose  $\mathcal{R}$  is a deterministic resource. We construct an environment  $\mathcal{Z}$  and pair of adversaries  $\mathcal{A}_{[2]}$  for which there are no simulators in the  $\mathcal{F}_{\text{coin}}^c$ -hybrid world.

Suppose protocol  $\pi$  is such that  $\mathcal{F}_{\text{coin}}^c \sqsubseteq_{\pi}^{CP} \mathcal{R}$  and for an execution of  $\pi$  we denote by  $Y$  the random variable describing its (common) output. Fix a random tape  $r_1$  for  $\pi_1$ . Then it must be that  $Y$  still has noticeable entropy even conditioned on the choice of  $r_1$ . If this were not the case then an adversary corrupting only  $\mathcal{P}_1$  and running  $\pi_1$  with tape  $r_1$  on his behalf could not be simulated since the output of the honest  $\mathcal{P}_2$  would always be fixed unlike an execution with  $\mathcal{F}_{\text{coin}}^c$  in the ideal world. More generally the number of random tapes  $r_2$  for  $\pi_2$  which cause  $Y$  to take on a fixed value is at most negligibly less than a  $1/2^c$  fraction of all possible values. Thus for any  $r_1$  it is easy to compute two tapes  $r_2^0$  and  $r_2^1$  which cause  $Y$  to take on two distinct values  $y^0$  and  $y^1$  respectively.

The adversaries each corrupt their respective player and run  $\pi$  on their behalf. Adversary  $\mathcal{A}_1$  uses random tape  $r_1$  and if  $\pi_1$  outputs  $y^0$  it outputs 0 otherwise it outputs 1. The environment provides  $\mathcal{A}_2$  with input a uniform random bit  $b$  and  $\mathcal{A}_2$  uses the random tape  $r_2^b$  for its copy of  $\pi_2$ . Thus by comparing the output of  $\mathcal{A}_1$  with the value of  $b$  over many executions the environment can distinguish the  $\mathcal{R}$ -hybrid world from any execution in a  $\mathcal{F}_{\text{coin}}^c$ -hybrid world. This is because the  $\mathcal{F}_{\text{coin}}^c$  functionality allows now communication between players and so a simulator for  $\mathcal{A}_2$  has no way to signal  $b$  to  $\mathcal{A}_1$ .  $\square$

### 3.3 Programmability.

We stated the goal of finding a CP-complete resource. Unfortunately, (unlike in models with monolithic adversaries) in the CP setting no single resource can be CP-complete. Instead we consider *parameterized sets* of resources. More formally we say that a set  $\mathcal{R} = \{\mathcal{R}_k\}_{k \in \mathcal{K}}$  of resources is *CP-complete* if for any functionality  $\mathcal{F}$ , there exists a resource  $\mathcal{R}_k \in \mathcal{R}$  such that  $\mathcal{F} \sqsubseteq^{CP} \mathcal{R}_k$ . Indeed later on, the mediator which we define is really a (CP-complete) *class* of resources. In our main theorem we only guarantee full security for the protocols we construct when the correct element of the mediator class is used. We argue that any completeness in the CP framework will require such a format.<sup>17</sup>

For index set  $\mathcal{K}$  let  $\mathcal{R}_{\mathcal{K}} := \{\mathcal{R}_k \mid k \in \mathcal{K}\}$  denote a set of resources parameterized by elements of  $\mathcal{K}$ . We call  $\mathcal{R}_{\mathcal{K}}$  a *programmable resource* if there exist  $k_1, k_2 \in \mathcal{K}$  such that  $\mathcal{R}_{k_1} \not\sqsubseteq^{CP} \mathcal{R}_{k_2}$ .

**Lemma 3.5.** *A class of resources is CP-complete only if it is programmable.*

<sup>16</sup>We opt for this unusually general formulation as the result does not require  $c = 1$  and so we can also capture ideal key exchange; another interesting functionality.

<sup>17</sup>To mitigate this we show that even if an arbitrary different resource is used (not necessarily from the mediator class) the protocol still attains a certain (GUC-like) security.

*Proof.* Let  $\mathcal{C}_{i,j}^1$  and  $\mathcal{C}_{i,j}^2$  denote the 1-bit and 2-bit (respectively) bounded collaborative channels between  $\mathsf{P}_i$  and  $\mathsf{P}_j$  (see the definition in Section 3.1). Let the class of resources  $\mathcal{R}_{\mathcal{K}}$  be CP-complete. Then there exists  $a, b \in \mathcal{K}$  and protocol  $\pi$  such that  $\mathcal{C}_{i,j}^2 \sqsubseteq_{\pi}^{CP} \mathcal{R}_b$  and  $\mathcal{C}_{i,j}^1 \sqsubseteq^{CP} \mathcal{R}_a$ .

Assume for a moment that  $\mathcal{R}_{\mathcal{K}}$  is not programmable. This implies that for some protocol  $\phi$  we have  $\mathcal{R}_b \sqsubseteq_{\phi}^{CP} \mathcal{R}_a$ . Consider the  $n$ -tuple of adversaries  $\mathsf{A}_{[n]}$  living in the  $\mathcal{R}_a$ -hybrid world which each corrupt their respective players and then run protocol (stack)  $\phi \circ \pi$  on the players' behalf using input supplied by the environment. In other words, the adversaries corrupt all players and then turn  $\mathcal{R}_a$  into  $\mathcal{R}_b$  using protocol  $\phi$  and then turn  $\mathcal{R}_b$  into  $\mathcal{C}_{i,j}^2$  using protocol  $\pi$ . The message transmitted from  $\mathsf{A}_i$  and  $\mathsf{A}_j$  (over  $\mathcal{C}_{i,j}^2$ ) is supplied to  $\mathsf{A}_i$  by the environment and is the output of  $\mathsf{A}_j$ .

We construct environment  $\mathcal{Z}$  for which no simulators exist in the  $\mathcal{C}_{i,j}^1$ -hybrid world (much as in the proof of Lemma 3.2). It selects a pair of random bits  $r \in \{0, 1\}^2$  and gives them to  $\mathsf{A}_i$  as input. Then it waits for  $\mathsf{A}_j$  to produce output  $r'$ . Finally  $\mathcal{Z}$  outputs 1 if and only if  $r = r'$ . Clearly in the  $\mathcal{C}_{i,j}^1$ -hybrid world the  $i^{\text{th}}$  and  $j^{\text{th}}$  adversary do not have enough bandwidth between them to simulate for this strategy which contradicts  $\mathcal{C}_{i,j}^1 \sqsubseteq^{CP} \mathcal{R}_a$ .  $\square$

## 4 GUC Fallback Security

Without any further requirements CP security as defined in Definition 2.1 can be easily achieved from an appropriate resource. Indeed, because the resource is completely trusted it could trivially be the functionality we are trying to compute. However such trust is a rare commodity and so one might ask for a better solution. To that end we add a second property which we call “fallback security”. The goal is to capture what kind of security remains if the protocol is run not with the resource it was designed for but with an arbitrary (potentially malicious) resource instead. Note that the trivial solution provides essentially no fallback security at all. However we will show, perhaps somewhat surprisingly, that in fact a very strong type of security can still be achieved; namely GUC-like realization.

**Definition 4.1** (CP-realization with GUC fallback). *For setup  $\bar{\mathcal{G}}$ , functionalities  $\mathcal{F}$  and  $\mathcal{R}$ , we say that a protocol  $\pi$  CP-realizes a functionality  $\mathcal{F}$  with GUC fallback in the  $\{\bar{\mathcal{G}}, \mathcal{R}\}$ -hybrid model if it has the following properties:*

CP SECURITY:  $\{\bar{\mathcal{G}}, \mathcal{F}\} \sqsubseteq_{\pi}^{CP} \{\bar{\mathcal{G}}, \mathcal{R}\}$

GUC FALLBACK: *For any efficient resource  $\mathcal{R}^*$ :  $\{\bar{\mathcal{G}}, \mathcal{R}_{\text{ins}}, \mathcal{F}\} \sqsubseteq_{\pi}^{CP} \{\bar{\mathcal{G}}, \mathcal{R}^*\}$ .*

Recall that by default the GUC plain model is equipped with  $[\mathcal{R}_{\text{ins}}]$ . Thus applying Lemma 2.5 and omitting the redundant  $[\mathcal{R}_{\text{ins}}]$  term we have that GUC fallback security directly implies:

$$\{[\bar{\mathcal{G}}], [\mathcal{F}]\} \sqsubseteq_{\pi}^{GUC} \{[\bar{\mathcal{G}}], [\mathcal{R}^*]\}.$$

Intuitively this means that  $\pi$  run in the presence of  $\mathcal{R}^*$  and arbitrarily coordinated adversaries still GUC realizes  $[\mathcal{F}]$ .

We note that as an alternative, by restricting the class of resources  $\mathcal{R}^*$  for which the fallback is desired one could in turn hope for stronger but still non-trivial fallback properties. This could reflect the real world settings where moderate guarantees about the behavior of the resource are given but it is still undesirable to completely trust the resource. In this sense the feasibility result in this work demonstrates that at the very least GUC fallback can be achieved even when no moderate guarantees of any type are made.

Next we provide a useful tool for proving GUC fallback.

**A “Dummy”-Resource Lemma.** We show that instead working with arbitrary resource  $\mathcal{R}^*$ , it suffices to prove that GUC fallback holds for a special “dummy” resource  $\mathcal{M}^*$ , which behaves as a forwarder between all parties and a dedicated adversary, e.g., the one with the smallest index.

More precisely the *dummy resource*, denoted by  $\mathcal{M}^*$ , is the  $n$ -party functionality behaving as follows:

- When  $\mathcal{M}^*$  receives a message by some party or adversary,  $\mathcal{M}^*$  forwards it to a default adversary, e.g., the adversary of the party with the smallest index (wlog assume that this is  $A_1$ )
- When  $\mathcal{M}^*$  receives a message of the form  $(\text{send}, m, ID)$  from  $A_1$ , where  $ID$  is the identifier of some party or adversary,  $\mathcal{M}^*$  sends  $m$  to the corresponding party or adversary, respectively.

As shown in the following lemma,  $\mathcal{M}^*$  capture the behavior of the worst possible resource  $\mathcal{R}^*$ . In particular, the following lemma, allows us to reduce the GUC-fallback property (of Definition 4.1) to CP security in the  $\mathcal{M}^*$ -hybrid model. In other words any statement about a monolithic adversary/simulator is translated to a statement in the  $\mathcal{M}^*$ -hybrid model, where the simulator for  $\text{Sim}_1$  plays the role of a coordinator among the simulators to allow the to mimic the behavior a monolithic simulator.

**Lemma 4.2.** *Let  $\mathcal{M}^*$  denote the “dummy” resource defined above. Moreover let  $\pi$  be a protocol,  $\bar{\mathcal{G}}$  be a shared functionality and  $\mathcal{F}$  be a CP functionality. Then for any efficiently computable resource  $\mathcal{R}^*$  it holds that:*

$$\{\bar{\mathcal{G}}, \mathcal{M}^*, \mathcal{F}\} \sqsubseteq_{\pi}^{CP} \{\bar{\mathcal{G}}, \mathcal{M}^*\} \implies \{\bar{\mathcal{G}}, \mathcal{R}_{\text{ins}}, \mathcal{F}\} \sqsubseteq_{\pi}^{CP} \{\bar{\mathcal{G}}, \mathcal{R}^*\}.$$

*Proof.* Let  $\mathcal{R}^*$  be some efficiently computable resource,  $\mathcal{D}^{\mathcal{R}^*}$  be the  $\mathcal{R}^*$ -hybrid dummy protocol and  $\mathcal{D}^{\mathcal{M}^*}$  be the dummy  $\mathcal{M}^*$ -hybrid protocol. One can easily verify the following two properties hold simultaneously:

1.  $\{\bar{\mathcal{G}}, \mathcal{M}^*\} \sqsubseteq_{\mathcal{D}^{\mathcal{R}^*}}^{CP} \{\bar{\mathcal{G}}, \mathcal{R}^*\}$
2.  $\{\bar{\mathcal{G}}, \mathcal{R}_{\text{ins}}, \mathcal{F}\} \sqsubseteq_{\mathcal{D}^{\mathcal{M}^*}}^{CP} \{\bar{\mathcal{G}}, \mathcal{M}^*, \mathcal{F}\}.$

Indeed, for property (1) the simulator  $S_1$  for  $A_1$  need simply simulate the behavior of  $\mathcal{R}^*$  (the assumption of efficient computability of  $\mathcal{R}^*$  ensures that  $S_1$  can do so). As  $\mathcal{M}^*$  is a forwarder between  $S_1$  and the parties, this setting is indistinguishable from the  $\mathcal{R}^*$ -hybrid setting. For property (2) all simulators use  $\mathcal{R}_{\text{ins}}$  to redirect any messages originally destined for  $\mathcal{M}^*$  to  $S_1$ . Moreover for any message of the form  $(\text{send}, m, ID)$  to be sent by  $A_1$  to  $\mathcal{M}^*$  the simulator  $S_1$  instead uses  $\mathcal{R}_{\text{ins}}$  to forward  $m$  directly to the party with the matching ID.

The above two properties applied in order together with Theorem 2.4 (the composition theorem) imply the lemma.<sup>18</sup>

$$\{\bar{\mathcal{G}}, \mathcal{M}^*, \mathcal{F}\} \sqsubseteq_{\pi}^{CP} \{\bar{\mathcal{G}}, \mathcal{M}^*\} \implies \{\bar{\mathcal{G}}, \mathcal{M}^*, \mathcal{F}\} \sqsubseteq_{\pi}^{CP} \{\bar{\mathcal{G}}, \mathcal{R}^*\} \tag{3}$$

$$\implies \{\bar{\mathcal{G}}, \mathcal{R}_{\text{ins}}, \mathcal{F}\} \sqsubseteq_{(\mathcal{D}^{\mathcal{M}^*})^{\pi}}^{CP} \{\bar{\mathcal{G}}, \mathcal{R}^*\} \tag{4}$$

$$\implies \{\bar{\mathcal{G}}, \mathcal{R}_{\text{ins}}, \mathcal{F}\} \sqsubseteq_{\pi}^{CP} \{\bar{\mathcal{G}}, \mathcal{R}^*\} \tag{5}$$

The third implication follows from the composition theorem applied to the protocol  $(\mathcal{D}^{\mathcal{M}^*})^{\pi}$  which forwards between  $\mathcal{Z}$  and  $\pi$  running in the  $\{\bar{\mathcal{G}}, \mathcal{R}^*\}$ -hybrid world (instead of between  $\mathcal{Z}$  and  $\mathcal{F}$  as would protocol  $\mathcal{D}^{\mathcal{M}^*}$ ). In other words, the composition implies  $(\mathcal{D}^{\mathcal{M}^*})^{\pi} = \pi$  in the  $\{\bar{\mathcal{G}}, \mathcal{R}^*\}$ -hybrid world.  $\square$

<sup>18</sup>Formally, we also make use of the convention of omitting dummy protocols from protocol call hierarchy (see Section 2). That is if  $\mathcal{D}^{\mathcal{R}^*}$  is the dummy protocol for  $\mathcal{R}^*$ , by convention  $\pi^{\mathcal{D}^{\mathcal{R}^*}} = \pi$ .

## 5 A General Feasibility Result

We are now ready to prove a general feasibility result. Roughly speaking we describe an (efficient) programmable resource  $\{\mathcal{M}_{\mathcal{F}}\}_{\mathcal{F} \in \{0,1\}^*}$ , called the *mediator*, parametrized by descriptions of functionalities, such that for any  $\mathcal{F}$  in a large class of functionality, we can design a protocol  $\pi$  (using setup  $\bar{\mathcal{G}}$ ) which CP-realizes  $\mathcal{F}$  with GUC fallback in the  $\{\bar{\mathcal{G}}, \mathcal{M}_{\mathcal{F}}\}$ -hybrid model.

**Output-Synchronized Functionality.** In contrast to previous frameworks, because we consider split simulators the environment  $\mathcal{Z}$  has an additional means for distinguishing between executions. Briefly,  $\mathcal{Z}$  can measure the amount of on-line synchronization that taking part in an execution provides to sets of adversaries. In contrast all actions taken by monolithic adversaries during an execution are already inherently perfectly synchronized so no such strategy exists in (G)UC frameworks.

A bit more formally suppose the  $n$ -party functionality  $\mathcal{F}$  takes one input from each player  $P_i$  and then produces some (global) output  $y$ . Moreover we want to realize  $\mathcal{F}$  with a protocol  $\pi$  which requires strictly more than one such round.<sup>19</sup> Then  $\mathcal{Z}$  can use the following fact to distinguish: it knows that in an execution of  $\pi$  the adversary  $A_j$  can not compute  $y$  in less than two rounds. However, in general the simulators  $\text{Sim}$  which only have access to  $\mathcal{F}$  may not be able to communicate to each other how many times they have been activated. In particular  $\text{Sim}_i$  cannot keep track of how often say  $\text{Sim}_j$  has been activated. So  $\text{Sim}_i$  has no way of knowing when to deliver the output  $y$  to  $\mathcal{Z}$ .

We address this by introducing the class of *output-synchronized* functionalities. For a arbitrary functionality  $\mathcal{F}$  we write  $\hat{\mathcal{F}}$  to denote the “output synchronized” version of  $\mathcal{F}$ . That is  $\hat{\mathcal{F}}$  consists of a wrapper (the *synchronizing shell*) and inner functionality  $\mathcal{F}$ . The synchronizing shell works as follows:

- On it’s first activation,  $\hat{\mathcal{F}}$  sends a request to one of the simulators (e.g. the one with the smallest ID) to acquire the index of the desired output round. Let  $R$  denote the response of this simulator (if no valid  $R$  is received then set  $R := 1$ ).
- Subsequently, all inputs are forwarded to  $\mathcal{F}$ .
- Outputs of  $\mathcal{F}$  are not immediately delivered to their recipient. Rather they are recorded and are given only *upon request* from the recipient and only after  $R$  subsequent *complete rounds* have been observed (each output-request which is issued before that is answered by a default message  $\perp$ ). By a complete round we mean a sequence of at least one activation per player in an arbitrary order.

We note that intuitively this modification provides minimal synchronization between adversaries. In particular only the output delivery is synchronized but surprisingly, they can not for example, tell at any given moment during their execution which round another adversary believes they are in. Nevertheless it turns out that output synchronization is a sufficient condition enabling CP secure realization. For the remaining on this section we shall assume that  $\mathcal{F}$  is an *output-synchronized* functionality, i.e, is of the form  $\hat{\mathcal{F}}'$  for some functionality  $\mathcal{F}'$ .

To formalize our main feasibility theorem:

**WELL-FORMED FUNCTIONALITY:** As in [12], due to technical limitations inherited from UC-type frameworks, we restrict ourselves to the class of *well-formed* functionalities[12], which

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<sup>19</sup>Observe that most general MPC protocols need more than one round for many interesting non-reactive functionalities.

intuitively includes all functionalities whose behavior does not depend on the identities of corrupted parties.

**ABORTING FUNCTIONALITY:** Consistent with existing literature on general secure computation tolerating a dishonest majority, we get security *with abort*. We capture this with the notion of an *aborting* functionality. These are such that they might accept a special input **ABRT**. Moreover, if any corrupted player provides input **ABRT** then the output of all players is also **ABRT**.

**SETUP OFF-LINE PROTOCOL:** A protocol which precedes all other computation and communication by it's only interaction with the setup  $\bar{\mathcal{G}}$  is called *setup off-line*.<sup>20</sup> Roughly speaking our construction has the mediator run a GUC protocol obviously but verifiably on behalf of the players. If the  $\pi$  were not setup off-line then it is unclear by whom the interaction with  $\bar{\mathcal{G}}$  could be performed. On the one hand players should remain oblivious to the messages of  $\pi$  (to obtain CP security) and the other hand the mediator should not be trusted to perform the interaction (so as to obtain GUC fallback).

**GUC-AUTHCOMPLETE SETUP:** We call a setup  $\bar{\mathcal{G}}$  (i.e. a CP shared functionality) *GUC-AuthComplete* if in the  $[\bar{\mathcal{G}}]$ -hybrid world:

1. There exists a setup off-line protocol which GUC realizes authenticated channels from insecure channels.
2. Every well-formed functionality can be GUC securely realized (in the standard GUC model which assumes authenticated channels) by a setup off-line protocol.

We note that one such setup is the Key-Registration with Knowledge (KRK) functionality of [11, 16] when viewed as a CP shared functionality (we refer to Section 5.3 for details). In particular the results of [16] imply Property (1) and the results of [11] imply Property (2).

Ideally, we would like to state our feasibility result for *any* (albeit efficient) functionality  $\mathcal{F}$ . More realistically we require  $\mathcal{F}$  some additional (standard) properties.

**Theorem 5.1** (General Feasibility Theorem). *Let  $\bar{\mathcal{G}}$  be a GUC-AuthComplete CP setup there exists a programmable resource  $\mathbf{M} = \{\mathbf{M}_{\mathcal{F}}\}$  such that for every well-formed aborting functionality  $\mathcal{F}$  there exists a protocol  $\pi$  which CP-realizes  $\mathcal{F}$  with GUC fallback in the  $\mathbf{M}_{\mathcal{F}}$ -hybrid model.*

We prove the theorem constructively, i.e., by describing an efficient compiler mapping a given aborting well-formed functionality  $\mathcal{F}$  to a protocol  $\text{CP}(\pi)$  and parameters for resource  $\mathbf{M}$ . We point out that although we have assumed that  $\mathcal{F}$  is output-synchronized, the GUC property holds, for the same protocol  $\text{CP}(\pi)$ , even for the non-synchronized functionality, i.e., the one that results by removing from  $\mathcal{F}$  the synchronizing shell. The reason is that in the GUC-fallback setting the simulators can synchronize when to produce output by using the insecure channels  $\mathcal{R}_{\text{ins}}^*$

The proof of the theorem proceeds in two steps:

1. We show how to obtain from a GUC-AuthComplete setup  $\bar{\mathcal{G}}$ , e.g., the KRK [11], a setup off-line protocol  $\pi$  which CP realizes  $\mathcal{F}$  using insecure channels (in subsection 5.1).
2. Then we show how to compile  $\pi$  into protocol  $\text{CP}(\pi)$  and resource  $\mathbf{M}_{\mathcal{F}}$  which CP realize  $\mathcal{F}$  with GUC fallback (in subsection 5.2).

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<sup>20</sup>We note that all known protocols satisfy this definition. In particular the protocols of [11, 16] are of this form.

## 5.1 Bootstrapping from GUC

We prove the following lemma.

**Lemma 5.2.** *Let  $\bar{\mathcal{G}}$  be a GUC-AuthComplete CP setup. Then for every wellformed aborting CP functionality  $\mathcal{F}$  there exist a setup off-line protocol  $\pi$  such that  $\{\bar{\mathcal{G}}, \mathcal{R}_{\text{ins}}, \mathcal{F}\} \sqsubseteq_{\pi}^{CP} \{\bar{\mathcal{G}}, \mathcal{R}_{\text{ins}}\}$ .*

*Proof.* Let  $[\mathcal{F}]$  be the GUC analogue<sup>21</sup> of  $\mathcal{F}$ . By assumption  $\bar{\mathcal{G}}$  is GUC-AuthComplete so there exists a GUC protocol  $\pi$  such that  $\{[\bar{\mathcal{G}}], [\mathcal{F}]\} \sqsubseteq_{\pi}^{GUC} [\bar{\mathcal{G}}]$ . As  $\bar{\mathcal{G}}$  is GUC-AuthComplete we can apply Lemma 2.6 to obtain the result.  $\square$

## 5.2 Adding Fallback Security

We prove the following lemma which states that if a protocol exists that CP realizes  $\mathcal{F}$  from insecure channels then there exists a protocol and resource which additionally have GUC fallback. The proof of Theorem 5.1 follows directly from this and Lemma 5.2.

**Lemma 5.3.** *Let  $\bar{\mathcal{G}}$  be a GUC-AuthComplete CP setup; let  $\mathcal{F}$  be a well-formed aborting functionality and let  $\pi$  be a setup off-line protocol such that  $\{\bar{\mathcal{G}}, \mathcal{R}_{\text{ins}}, \mathcal{F}\} \sqsubseteq_{\pi}^{CP} \{\bar{\mathcal{G}}, \mathcal{R}_{\text{ins}}\}$ .*

*Then there exists an efficient resource  $M_{\mathcal{F}}$  (called the “mediator”) and protocol  $CP(\pi)$  such that  $CP(\pi)$  CP realizes  $\mathcal{F}$  with GUC fallback in the  $M_{\mathcal{F}}$ -hybrid model.*

**Proof Idea.** The proof is constructive, i.e., we show how to construct  $CP(\pi)$  and  $M_{\mathcal{F}}$ , and somewhat involved; it is inspired by the protocol of [3]. On the highest level the idea is to have the mediator  $M_{\mathcal{F}}$  emulate an execution  $\pi$  “in his head” such that the players are oblivious to everything but their input and output of  $\pi$ . Intuitively this guarantees the CP realization property of Definition 4.1.

To obtain the GUC fallback property we need to make the following modifications to this approach. For each  $i \in [n]$  the state of the emulated  $\pi_i$  is *shared* between  $P_i$  and  $M_{\mathcal{F}}$  such that  $M_{\mathcal{F}}$  can not alter it without the help of  $P_i$  yet both parties learn nothing about the actual value of the state. For this purpose we describe a pair of 2-party SFE’s run between  $P_i$  and  $M_{\mathcal{F}}$  which allow for state of  $\pi_i$  to be updated as dictated by an honest execution of  $\pi$ . Intuitively it is the hiding of the states from  $M_{\mathcal{F}}$  and the security of the SFE’s which ensure GUC fallback.

While enjoying a significantly stronger security notion, our compiler is also conceptually simpler than the one in [3]. This stems from our assumption that the input protocol  $\pi$  operates over a *network of insecure channels* rather than the broadcast channel used in [3]. As a result, (1) our compiler does not need to worry about the parties authenticating their messages, as this is taken care of by  $\pi$ , and (2) we do not need specifically describe a “mediated” broadcast protocol as in [3].

**Assumptions and Constraints.** For simplicity we make the following assumptions.

NON-REACTIVE  $\mathcal{F}$ : We restrict ourselves to the case of non-reactive functionalities  $\mathcal{F}$ , aka Secure Function Evaluation (SFE); however at the cost of more involved notation, the same methods can be extended to the case of reactive functionalities.

Without loss of generality we also assume the following.

---

<sup>21</sup>Recall that in Section 3, for a CP functionality  $\mathcal{F}$  we defined  $[\mathcal{F}]$  to be the monolithic extension of a CP functionality.

**FIXED ROUNDS:** We assume that  $\pi$  consists of a sequence of rounds. In each round, every player sends a fixed length message (from a public domain  $\mathcal{M}$ ) to every other player. The rounds are *non-overlapping*, in the sense that messages which a party sends in some round (even when he is corrupted) depend on the party’s input, randomness, and the messages that were sent to this party *up to that round*.<sup>22</sup> Such a non-overlapping rounds structure can be obtained by known protocols, even in the asynchronous setting, by using synchronization techniques from [30, 29]. Furthermore, we assume that the number of rounds is fixed<sup>23</sup> and public, and that every party receives its  $\pi$  output in the same round. We note that most known general MPC protocols either already satisfy this and the next property, or can be trivially modified to do so. Specifically all GUC protocols of [11, 16] (which are presumably the protocols that will be used to instantiate our construction as in for example Corollary 5.6) satisfy this and the next property.

**BLACK-BOX SIMULATION:** We assume that  $\pi$  CP realizes  $\mathcal{F}$  with black-box simulation. We point out that this is already implied by Definition 2.1.

**COMMITMENT PROTOCOL:** We use a GUC-secure commitment scheme  $(\text{Com}, \text{Dec})$  with the same message space  $\mathcal{M}$  as  $\pi$ . The scheme’s existence follows from the facts that  $\bar{\mathcal{G}}$  is GUC-AuthComplete and that the commitment functionality is well- formed and aborting. In particular [11] provides a concrete construction.

**We sketch the remainder of the proof which is detailed in Sections 5.2.1 and 5.2.2:**

**IN SECTION 5.2.1:** First, we describe how to share the state of  $\pi_i$  between a player and the mediator. Next we describe two useful programmable functionalities  $\mathcal{F}_{\text{inp}} = \{\mathcal{F}_{\text{inp}}^{(\pi)}(A, B)\}_{\pi \in \{0,1\}^*}$  and  $\mathcal{F}_{\text{comp}} = \{\mathcal{F}_{\text{comp}}^{(\pi)}(A, B)\}_{\pi \in \{0,1\}^*}$ .<sup>24</sup> These specify how from (a sharing of) some valid  $\pi$ -state for a party  $P_i$ , the mediator and  $P_i$  can compute a sharing of  $P_i$ ’s next message, and thus an updated  $\pi$ -state from  $P_i$ . To avoid over-complicated notation, wherever it is clear from the context or redundant we might omit the protocol from the notation of the above functionalities, i.e., we write  $\mathcal{F}_{\text{comp}}$  (resp.  $\mathcal{F}_{\text{inp}}$ ) instead of  $\mathcal{F}_{\text{comp}}^{(\pi)}$  (resp.  $\mathcal{F}_{\text{inp}}^{(\pi)}$ ). In order to achieve the GUC fallback, the mediator should not be allowed to see the parties’ shares of the shared protocol states. Therefore, instead of invoking the functionality  $\mathcal{F}_{\text{inp}}$  and  $\mathcal{F}_{\text{comp}}$ , the mediator and  $P_i$  execute a two-party protocol implementing the corresponding functionality. In fact, as we shall show, the code of  $P_i$  in the compiled protocol  $\text{CP}(\pi)$  consists of playing as Bob in the evaluations of these two-party SFE’s.

**IN SECTION 5.2.2:** As soon as we specify the code of the mediator and of the parties, the proof proceeds along the lines of the proof of [3]: We analyze the security of the protocol in the  $\{\mathcal{F}_{\text{inp}}, \mathcal{F}_{\text{comp}}\}$ -hybrid world, and use composition to argue that replacing the hybrids by the corresponding protocols does not compromise security. The main technical obstacle in this approach is that, similarly to UC and GUC, our framework does not specify how different functionalities could communicate

<sup>22</sup>The adversary is rushing, hence, messages of corrupted parties in any round might depend on the messages which honest parties send in that round, *but not on messages that honest parties prepare for future rounds*.

<sup>23</sup>More generally, at the cost of elegance, we could have assumed only that the distribution describing the number of rounds in an honest execution of  $\pi$  be efficiently sampleable and independent of all inputs used by the players.

<sup>24</sup>We use Alice ( $A$ ) and Bob ( $B$ ), and not indices  $i, j \in [n]$ , for the parties executing  $\mathcal{F}_{\text{inp}}$  and  $\mathcal{F}_{\text{comp}}$  to avoid confusion, as we intend to have the mediator play the role of Alice. Furthermore, for notational simplicity we will often omit the superscript  $(\pi)$  from this notation.

with each-other. Hence, it is not straight-forward how to make the argument in the  $\{\mathcal{F}_{\text{inp}}, \mathcal{F}_{\text{comp}}\}$ -hybrid world, as this would require the mediator to communicate with the hybrids. To overcome this difficulty, we consider the model where the mediator is replaced by an extra party  $P_M \notin [n]$ ;  $P_M$  executes the code of the mediator and shares an ideal bidirectional secure channel with every  $P_i$  (i.e., we have a star network of ideal secure channels where  $P_M$  is in the middle of the star). Because the mediator is a functionality and cannot be corrupted, we will only consider in the security statements for the case when  $P_M$  is honest. However, in order to proof CP realization with GUC fallback, we will consider two versions/programs of  $P_M$ : the first one, denoted as  $P_{M_{\mathcal{F}}}$ , will behave as the mediator  $M_{\mathcal{F}}$  and we will show that it allows for CP-realizing the functionality  $\mathcal{F}$ . The second one, denoted as  $P_{M^*}$ , will behave as the dummy mediator  $M^*$ , will allow us to achieve the security required for the GUC fallback.

### 5.2.1 A Detailed Description of $M_{\mathcal{F}}$ and $\text{CP}(\pi)$ .

Intuitively the mediator  $M_{\mathcal{F}}$  and the compiled protocol  $\text{CP}(\pi)$  use the protocols  $\pi_{\text{inp}}$  and  $\pi_{\text{comp}}$  that securely implement the functionalities  $\mathcal{F}_{\text{inp}}$  and  $\mathcal{F}_{\text{comp}}$ , respectively, as follows.

**INIT:** Initially, mediator  $M_{\mathcal{F}}$  sets the round index to  $\rho := 0$ .

**INPUTS:** The first interaction between a player  $P_i$  and  $M_{\mathcal{F}}$  involves an invocation of  $\pi_{\text{inp}}$ . The purpose is to fix  $P_i$ 's input  $x_i$  and a random tape  $r_i$  used in the emulation of  $\pi_i$ . The output of  $\pi_{\text{inp}}$  for the mediator is commitments to  $x_i$  and  $r_i$  while  $P_i$  receives the matching decommitments. As soon as every party has committed to his input and randomness,  $M_{\mathcal{F}}$  augments the round index to  $\rho := 1$ .

**COMPUTATION & OUTPUTS:** All subsequent interactions between  $P_i$  and  $M_{\mathcal{F}}$  consist of invocations of  $\pi_{\text{comp}}$  which takes input the current (shared) state of  $\pi_i$  together with an  $(n - 1)$ -tuple of messages received by  $\pi_i$  during the current round. Initially  $\mathcal{F}_{\text{comp}}$  checks that all inputs match and are of a valid form. Next it computes the next state of  $\pi_i$  together with an  $(n - 1)$ -tuple of messages out produced by  $\pi_i$  for use in the next round  $\rho + 1$ .  $\mathcal{F}_{\text{comp}}$  outputs the updated shared state and, to  $M_{\mathcal{F}}$ , the messages out. Finally, if  $\rho$  is the last round of  $\pi$  then  $P_i$  also receives its output of the computation (and otherwise it receive a default message  $\perp$ ).

For any value of  $\rho$  which is not the last round-index, as soon as the mediator has completed one computation of  $\pi_{\text{comp}}$  with each player,  $M_{\mathcal{F}}$  increments  $\rho := \rho + 1$ .

In the following, we describe in detail the functionalities  $\mathcal{F}_{\text{inp}}$  and  $\mathcal{F}_{\text{comp}}$  (which implicitly depend on  $n$  and the code of  $\pi$ ). As already mentioned, these are functionalities between two parties, Alice and Bob, where the role of Alice will be player by the mediator, and the role of Bob will be played by each of the parties in  $[n]$ . Recall that we denote by  $\mathcal{M}$  the message-space of protocol  $\pi$ . We assume that  $\mathcal{M}$  is equipped with a special abort message **ABRT**.

We begin the detailed description with some language and notation for working with the state of party  $P_i$  in protocol  $\pi_i$  (for simplicity, we denote  $P_i$ 's code in  $\pi$  as  $\pi_i$ ).

**Shared Protocol State.** In a nutshell we model a shared state of a protocol as a pair of matching vectors of commitments/decommitments to the view of an execution. More precisely, recall that  $\pi$  has a fixed number of rounds  $\text{rmax} \in \mathbb{N}$ , during each of which each player sends  $(n - 1)$  messages from message space  $\mathcal{M}$ , one to each of the other player. We call an  $(n - 1)$ -tuple of messages  $\vec{m}$  (containing one component either per sender or per receiver) a *message burst*. I.e. in each round each player receives a message burst and sends a message burst.<sup>25</sup> At any point of the

<sup>25</sup>For the first round the received message bursts can take on arbitrary fixed default value.



execution of protocol  $\pi$ , the received message bursts, together with the random tape  $r_i$  and input  $x_i$  completely determine the state of  $\pi_i$ . Thus we model the *state of  $\pi_i$*  as a quadruple of the form  $\Pi_i = (x_i, r_i, M_i, \rho)$  where  $\rho$  is the index of the current round, and  $M_i$  is an  $\mathbf{rmax} \times (n-1)$  matrix with rows  $\vec{m}_a$  such that for all  $a \in [\mathbf{rmax}]$  and  $b \in [n-1]$  the entry  $m_{a,b} \in \mathcal{M} \cup \{\perp\}$ . The goal of  $\mathcal{F}_{\text{inp}}$  will be to initialize  $\Pi_i$  and the goal of  $\mathcal{F}_{\text{comp}}$  will be to iteratively and obviously fill in the entries of  $M_i$ .

We call a state  $\Pi_i$  *sound for round  $\mathbf{r}$*  (where  $0 \leq \mathbf{r} \leq \mathbf{rmax}$ ) if:

1.  $\rho = \mathbf{r}$ .
2. For all  $j \in [\rho]$  each message burst  $\vec{m}_j \in \mathcal{M}^{n-1}$ .
3. For all  $j \in \{\rho+1, \dots, \mathbf{rmax}\}$  each message burst  $\vec{m}_j = \perp^{n-1}$ .

A *shared state* is a pair of quadruples  $(\vec{C}_i, \vec{D}_i)$  where  $\vec{C}_i$  maintained by  $P_i$  and  $\vec{D}_i$  is maintained by  $M_{\mathcal{F}}$ . Mediator  $M_{\mathcal{F}}$  holds commitments to  $x_i$  and  $r_i$ , while commitments to  $M_i$  and  $\rho$  are held by  $P_i$  (and vice versa for the corresponding decommitments).<sup>26</sup> In symbols  $P_i$  maintains the quadruple  $\vec{C}_i = [\text{Dec}(x_i), \text{Dec}(r_i), \text{Com}(\rho), \text{Com}(M_i),]$  while  $M_{\mathcal{F}}$  maintains  $\vec{D}_i = [\text{Com}(x_i), \text{Com}(r_i), \text{Dec}(\rho), \text{Dec}(M_i)]$ . We call a shared state  $(\vec{C}_i, \vec{D}_i)$  *valid for round  $\rho$*  if:

1. The matching components of  $\vec{C}_i$  and  $\vec{D}_i$  are valid commitment/decommitments pairs.
2. Let  $\Pi_i$  be the vector of decommitted values. Then  $\Pi_i$  is sound for round  $\rho$ .

**Functionality  $\mathcal{F}_{\text{inp}}(A, B)$ .** The goal of this functionality is to prepare a shared state of  $\pi_i$  between  $P_i$  (playing as Bob) and  $M_{\mathcal{F}}$  (plying as Alice). This includes fixing an input and random tape, initializing all incoming message bursts to a default value and then committing to the state using a GUC commitment protocol ( $\text{Com}, \text{Dec}$ ).

For sake of simplicity, we describe the functionality  $\mathcal{F}_{\text{inp}}$  (see Figure 4) in a form of a function. A CP functionality can be trivially obtained from our description, by considering the corresponding SFE functionality in the GUC framework [8, 11] and modifying it by applying the transformation  $[\cdot]$  along the lines of Section 2.4.1.

Functionality $\mathcal{F}_{\text{inp}}(A, B)$
<p>INPUT: This functionality takes input <math>x</math> from <math>B</math>.</p> <p>COMPUTATION:</p> <ol style="list-style-type: none"> <li>1. If <math>x</math> is not a valid input to <math>\pi_i</math> then <math>\mathcal{F}_{\text{comp}}</math> sets <math>(x, r) := (0, 0)</math>. Otherwise <math>\mathcal{F}_{\text{inp}}(i)</math> samples a fresh random tape <math>r</math>. Let <math>\perp</math> be the <math>\mathbf{rmax} \times (n-1)</math> matrix with all <math>\perp</math> entries.</li> <li>2. Set vector <math>\vec{C} = (\text{Dec}(x), \text{Dec}(r), \text{Dec}(1), \text{Com}(\perp))</math>.</li> <li>3. Set vector <math>\vec{D} = (\text{Com}(x), \text{Com}(r), \text{Com}(1), \text{Dec}(\perp))</math>.</li> </ol> <p>OUTPUT: Party <math>A</math> receives <math>\vec{D}</math> while <math>B</math> is given <math>\vec{C}</math>.</p>

Figure 4: A detailed description of functionality  $\mathcal{F}_{\text{inp}}(A, B)$ .

**Functionality  $\mathcal{F}_{\text{comp}}(A, B)$ .** This functionality is accessed by  $P_i$  (playing as Bob) and  $M_{\mathcal{F}}$  (playing as Alice). Its goal it to update the state of  $\pi_i$  and compute an outgoing message burst for the next round.

In more detail it takes input a shared state for  $\pi_i$  and, from  $M_{\mathcal{F}}$ , an (incoming) messages burst  $\vec{m}$  received by  $\pi_i$  during that round. The functionality checks that the shared state is valid for

<sup>26</sup>The commitment to matrix  $M_i$  is assumed to work component wise with the components arranged in a canonical order. For the sake of readability we use matrix notation.

some  $\rho \in [\mathbf{rmax}]$ . If not or if the state or  $\vec{m}$  contain an ABRT message then  $\mathcal{F}_{\text{comp}}$  set's the outgoing message burst to  $\{\text{ABRT}\}^{n-1}$ . Otherwise  $\mathcal{F}_{\text{comp}}$  updates the state with the new message tuples. If  $\rho = \mathbf{rmax}$  then  $\mathcal{F}_{\text{comp}}$  computes and sets  $y$  equal to the output of  $\pi_i$ . Otherwise it sets  $y$  to a default value. Finally,  $\mathcal{F}_{\text{comp}}$  outputs an updated sharing of the state to the parties, as well as  $y$  for  $P_i$  and the outgoing message burst for round  $\rho + 1$  to  $M_{\mathcal{F}}$ .

For a detailed description of  $\mathcal{F}_{\text{comp}}(A, B)$  see Figure 5. For simplicity we describe  $\mathcal{F}_{\text{inp}}$  in form of a function; as in the case of  $\mathcal{F}_{\text{inp}}$ , the actual functionality can be derived by taking the corresponding GUC functionality and modifying it along the lines of Section 2.4.1.

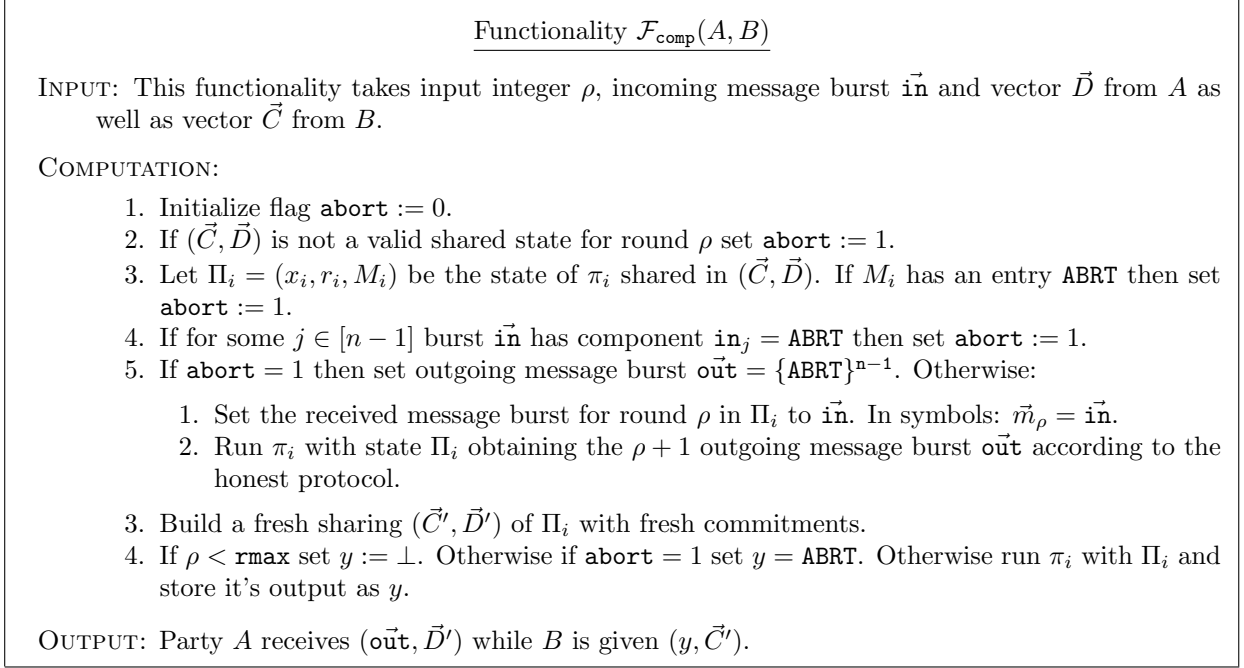


Figure 5: A detailed description of functionality  $\mathcal{F}_{\text{comp}}(A, B)$ .

*Remark 2* (Minimal Signaling by Abort). The specification of  $\mathcal{F}_{\text{comp}}$  ensure that by making the protocol abort, an adversary might signal no more than one bit to other adversaries. In particular, no adversary (other than the one that caused the abort) is informed about when or by whom the message that caused the abort was sent. This is consistent with other recent results in collusion-free computation [26, 3].

**The protocols  $\pi_{\text{inp}}(A, B)$  and  $\pi_{\text{comp}}(A, B)$ :** As already mentioned, the mediator  $M_{\mathcal{F}}$  and the parties in the compiled protocol  $\text{CP}(\pi)$  do not have access to the two-party functionalities  $\mathcal{F}_{\text{inp}}$  and  $\mathcal{F}_{\text{comp}}$ . Rather the behavior of these functionalities is emulated by corresponding two-party protocols  $\pi_{\text{inp}}$  and  $\pi_{\text{comp}}$ , which exchange messages with the mediator. In fact, one can verify that for CP-realizing  $\mathcal{F}_{\text{inp}}$  and  $\mathcal{F}_{\text{comp}}$  when ideally secure communication channels among the parties are assumed,<sup>27</sup> we can use protocols that GUC realize the corresponding GUC functionalities  $[\mathcal{F}_{\text{inp}}]$  and  $[\mathcal{F}_{\text{comp}}]$ , as long as these protocols are setup-off-line (Lemma Lemma 2.6). Such protocols are described in [11]. As syntactic sugar, we denote by  $\pi_{\text{inp}}^A$  and  $\pi_{\text{inp}}^B$  the code of Alice and Bob in  $\pi_{\text{inp}}$ , respectively; analogously we define  $\pi_{\text{comp}}^A$  and  $\pi_{\text{comp}}^B$ .

<sup>27</sup>By ideally secure channels we mean channels that resemble the communication means of the interface between the parties and the mediator. We refer to the following section for a detailed description.

**The Mediator  $M_{\mathcal{F}}$ .** We now turn to describing the mediator  $M_{\mathcal{F}}$ . In a nutshell, it's role is to emulate an execution of  $\pi$ . This is done by maintaining an  $n$ -tuple of shares of states for  $\pi$  (one per player), a counter  $\rho$  to keep track of the current round in the emulation, and two  $n$ -tuple of message bursts  $\vec{m}_i$  which contain the messages exchanged during the current round.

Initially  $\rho := 0$ . The mediator can perform two types of operations. Upon request by a player  $P_i$  the mediator engages with  $P_i$  into an execution of  $\pi_{\text{inp}}(M_{\mathcal{F}}, P_i)$  for evaluating  $\mathcal{F}_{\text{inp}}(M_{\mathcal{F}}, P_i)$  to initialize  $P_i$ 's state. This is done exactly once for each  $i \in [n]$ . Once all states have been initialized  $M_{\mathcal{F}}$  sets  $\rho := 1$ . The second operation (also upon request by some  $P_i$ ) consists of invocation of protocol  $\pi_{\text{comp}}(M_{\mathcal{F}}, P_i)$  for evaluating  $\mathcal{F}_{\text{comp}}(M_{\mathcal{F}}, P_i)$  on the current (sharing of) the state. These calls are performed upon any request by any player unless the state of that player has not been initialized yet. In particular, multiple calls to  $\pi_{\text{comp}}(M_{\mathcal{F}}, P_i)$  for the same  $P_i$  may be made *sequentially* during any given round –though only one is needed per round (for details on why accepting multiple computation-requests per round is useful see Remark 3). More precisely, during each round of the emulation the mediator makes at least one call to  $\pi_{\text{comp}}(M_{\mathcal{F}}, P_i)$  for every value of  $i \in [n]$ . Once this is done  $M_{\mathcal{F}}$  increments  $\rho := \rho + 1$ .

For a formal description of the mediator  $M_{\mathcal{F}}$  see Figure 6.

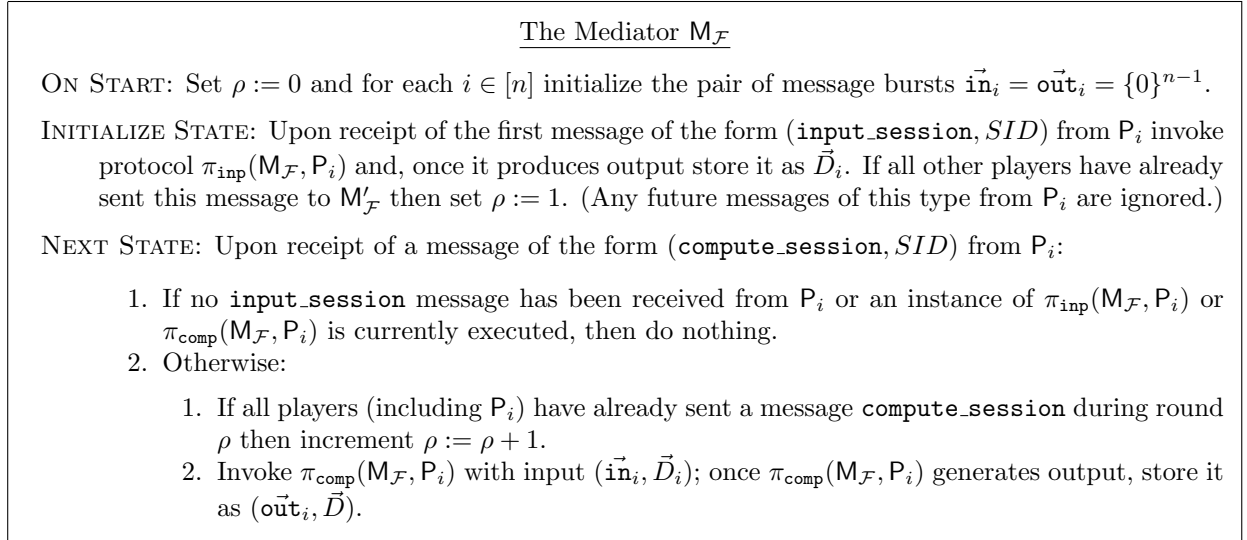


Figure 6: A detailed description of the mediator  $M_{\mathcal{F}}$ .

*Remark 3 (Round Obliviousness).* As mentioned in Section 4, the functionality  $\mathcal{F}$  which we compute is output-synchronized. This implies that it allows the simulators to ensure that they produce their output to  $\mathcal{Z}$  only when the last (simulated) round kicks in. However, the simulators are not aware of the following events: (1) every party has been activated for giving input, and (2) the same simulator is activated multiple times in the same round. For this reason, we have designed the mediator to hide this information from the adversaries as well. In particular, whenever the mediator receives a `compute_session` request from some  $P_i$ , he invokes the protocol  $\pi_{\text{comp}}(M_{\mathcal{F}}, P_i)$  for updating the state, irrespective of whether  $P_i$ 's state for this round has already been updated (by another invocation of  $\pi_{\text{comp}}(M_{\mathcal{F}}, P_i)$ ). By definition of  $\mathcal{F}_{\text{comp}}$ , if  $P_i$ 's state for the current round has already been computed, it will not be changed, but a new sharing of this state will be computed to hide from the adversaries this information.

**The Protocol  $\text{CP}(\pi)$ .** Having described the code of  $M_{\mathcal{F}}$ , the protocol  $\text{CP}(\pi)$  can be described in a straight-forward manner. In a nutshell, the code  $\text{CP}_i(\pi)$  of each party  $P_i$  is described as follows:

whenever  $M_{\mathcal{F}}$  invokes any of the protocols  $\pi_{\text{inp}}(M_{\mathcal{F}}, P_i)$  or  $\pi_{\text{comp}}(M_{\mathcal{F}}, P_i)$ , the party  $P_i$  executes his part (i.e., Bob's role) of that protocol. More precisely,  $P_i$  receives input  $x_i$  from the environment  $\mathcal{Z}$  and maintains a share  $\vec{C}$  of the state of  $\pi_i$ . The vector  $\vec{C}$  is initialized with an invocation of  $\pi_{\text{inp}}(M_{\mathcal{F}}, P_i)$  and updated via multiple (sequential) invocations of  $\pi_{\text{comp}}(M_{\mathcal{F}}, P_i)$ . Eventually  $\pi_{\text{comp}}$  returns the output of  $\pi_i$  which is passed back to  $\mathcal{Z}$ . For a formal description see Figure 7.

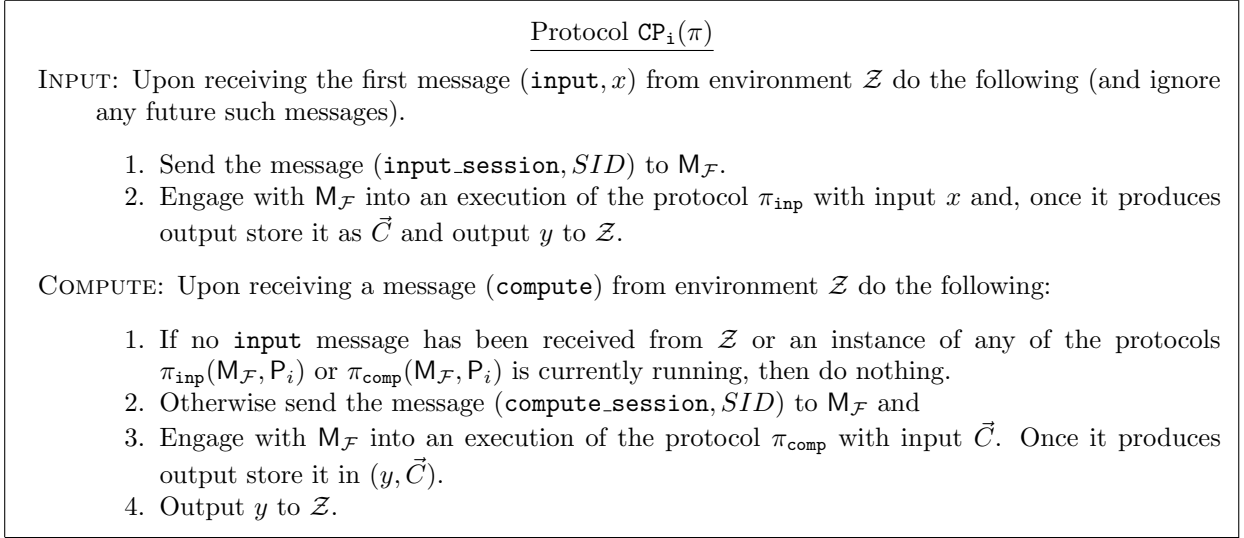


Figure 7: A detailed description of protocol  $\text{CP}_i(\pi)$ .

### 5.2.2 Completing the Proof of Lemma 5.3

To complete the proof, we show for the above described mediator  $M_{\mathcal{F}}$  and protocol  $\text{CP}(\pi)$ , the statement of Lemma 5.3 holds. To help the reader we give an outline of the rest of proof: We consider a world where the mediator is replaced by an extra party  $P_M \notin [n]$ .  $P_M$  might be programmed in one of two different manners (depending on the program of  $P_M$  we get CP or GUC-fallback security): in one version, denoted as  $P_{M_{\mathcal{F}}}$ , he executes exactly the code of the mediator  $M_{\mathcal{F}}$ , whereas in the other version, denoted as  $P_{M^*}$ , the code of the dummy mediator  $M^*$  (see Section 4).  $P_M$  communicates with the parties over bi-directional ideal communication channels  $\mathcal{F}_{\text{ideal}}$ . These are functionalities that resemble the interaction of the mediator with the party, e.g., in  $\mathcal{F}_{\text{ideal}}(P_M, P_i)$ ,  $P_M$  might send a message which is immediately delivered to  $P_i$ . Furthermore,  $\mathcal{F}_{\text{ideal}}$  allows  $P_M$  to exchange messages with the adversaries in  $A_{[n]}$  (note that  $P_{M_{\mathcal{F}}}$  will never use this additional functionality, but  $P_{M^*}$  will). We denote the star-network of such ideal channels where  $P_M$  sits in the middle as  $\mathcal{F}_{\text{ideal}}^* = \{\mathcal{F}_{\text{ideal}}(P_M, P_i), \mathcal{F}_{\text{ideal}}(P_i, P_M) \mid i \in [n]\}$ . Apart from this star-network there is no other communication resource among the parties.

Denote by  $\pi^*$  the  $(n+1)$ -party protocol, i.e., among the parties in  $[n] \cup \{P_M\}$ , in which  $P_M$  executes his program (i.e., either  $P_{M_{\mathcal{F}}}$  or  $P_{M^*}$ ) and for each  $i \in [n]$  party  $P_i$  executes his code from  $\text{CP}_i(\pi)$ , where the interaction with the resource  $M_{\mathcal{F}}$  or  $M^*$  is replaced by interaction with  $P_M$ . We compare the execution of  $\pi^*$  with the one of  $\text{CP}(\pi)$  *conditioned on  $P_M$  being honest*, and show that:

1. When  $P_M = P_{M_{\mathcal{F}}}$ : if the functionality  $\mathcal{F}$  (among the parties in  $[n]$ ) is CP realized from  $\pi^*$  then it is also CP-realized from  $\text{CP}(\pi)$  (in the  $M_{\mathcal{F}}$ -hybrid model)

<sup>28</sup>In slight abuse of notation we use  $\text{CP}(\pi)$  for denoting both the  $M^*$ -hybrid and the  $M_{\mathcal{F}}$ -hybrid protocol.

2. When  $P_M = P_{M^*}$ : if the functionality  $\{\mathcal{F}, \mathcal{R}_{\text{ins}}\}$  is CP-realized from  $\pi^*$  then it is also CP realized from  $\text{CP}(\pi)$  (in the  $M^*$ -hybrid model)

Next, we show that

3. When  $P_M = P_{M_{\mathcal{F}}}$ : the protocol  $\pi^*$  CP-realizes the functionality  $\mathcal{F}$ ; this together with Statement 1 imply that the protocol  $\text{CP}(\pi)$  CP-realizes  $\mathcal{F}$  in the  $M_{\mathcal{F}}$ -hybrid model.
4. When  $P_M = P_{M^*}$ : the protocol  $\pi^*$  CP securely realizes the functionality  $\{\mathcal{F}, \mathcal{R}_{\text{ins}}\}$ ; this together with Statement 2 imply that the protocol  $\text{CP}(\pi)$  CP-realizes  $\{\mathcal{F}, \mathcal{R}_{\text{ins}}\}$  in the  $M^*$ -hybrid model (GUC fallback).

Steps 3 and 4 are first proved for the setting where  $\pi^*$  is executed in the  $\{\mathcal{F}_{\text{inp}}, \mathcal{F}_{\text{comp}}\}$ -hybrid model (i.e., invocations of  $\pi_{\text{inp}}$  and  $\pi_{\text{comp}}$  are replaced by calls to  $\mathcal{F}_{\text{inp}}$  and  $\mathcal{F}_{\text{comp}}$ , respectively). The actual statements are then obtained by applying the composition theorem.

In the remaining of this section we prove the above steps.

**Steps 1 and 2:** The first two steps are relatively easy. Indeed, when  $P_M = P_{M_{\mathcal{F}}}$  then the environment witnesses essentially the same interaction as in the case where  $M_{\mathcal{F}}$  is playing instead of  $P_M$  (recall that we assume  $P_M$  to be honest). The same holds for  $P_M = P_{M^*}$  and  $M^*$ . We only need to take care of the following (technical) issue: In the execution of  $\pi^*$  we have one more party and adversary than in the execution of  $\text{CP}(\pi)$ . We argue that this is not a problem: indeed, one can consider the extra party  $P_M$  also in the  $M_{\mathcal{F}}/M^*$ -hybrid world, but have him do nothing, i.e., ignore all inputs from  $\mathcal{Z}$ . As, by assumption,  $P_M$  is honest and  $\mathcal{F}_{\text{ideal}}^*$  does not interact with  $P_M$ 's adversary, such an extension cannot change  $\mathcal{Z}$ 's distinguishing advantage.

**Step 3** We next show that when  $P_M = P_{M_{\mathcal{F}}}$  then the protocol  $\pi^*$  CP-securely realizes the functionality  $\mathcal{F}$ . We show this statement in two sub-steps:

**Sub-step 3.1.** We consider the protocol  $\pi_{\mathcal{F}_{\text{inp}}^*, \mathcal{F}_{\text{comp}}^*, \mathcal{F}_{\text{ideal}}^*}^*$  which works as follows:  $\pi_{\mathcal{F}_{\text{inp}}^*, \mathcal{F}_{\text{comp}}^*, \mathcal{F}_{\text{ideal}}^*}^*$  is a version of  $\pi^*$  where calls to protocols  $\pi_{\text{inp}}$  and  $\pi_{\text{comp}}$  are replaced by calls to the functionalities  $\mathcal{F}_{\text{inp}}$  and  $\mathcal{F}_{\text{comp}}$ . As syntactic sugar we denote the star-networks of such functionalities, where  $P_M$  sits in the middle, by  $\mathcal{F}_{\text{inp}}^* = \{\mathcal{F}_{\text{inp}}(P_M, P_i) \mid i \in [n]\}$  and  $\mathcal{F}_{\text{comp}}^* = \{\mathcal{F}_{\text{comp}}(P_M, P_i) \mid i \in [n]\}$

**Claim 5.4.** *Assuming  $P_M = P_{M_{\mathcal{F}}}$  is honest, the protocol  $\pi_{\mathcal{F}_{\text{inp}}^*, \mathcal{F}_{\text{comp}}^*, \mathcal{F}_{\text{ideal}}^*}^*$  CP-securely realizes the functionality  $\mathcal{F}$ .*

*Proof.* We show the claim for the notion of SCP security, i.e., where the environment invokes only one instance of  $\pi_{\mathcal{F}_{\text{inp}}^*, \mathcal{F}_{\text{comp}}^*, \mathcal{F}_{\text{ideal}}^*}^*$  and the assumed adversaries are “dummy”. The idea for the simulation is the following: Before the output is generated from  $\mathcal{F}$ , the only thing that any adversary  $A_i$  of some corrupted  $P_i$  sees is the commitments to the state that are output from  $\mathcal{F}_{\text{inp}}$  and  $\mathcal{F}_{\text{comp}}$  and the de-commitment information on the input  $x$ . Because these commitments are generate with independent randomness, they can be easily simulated. In the following we sketch how the simulator  $\text{Sim}_i$  for  $A_i$  works. The program of all simulators is the same, except from  $\text{Sim}_1$  who, in addition to what the other simulators do, starts of by sending the maximum round  $\text{rmax}$  to the (synchronizing shell of) the functionality  $\mathcal{F}$ . The simulator  $\text{Sim}_i$  works as follows (we only consider the case where  $P_i$  is corrupted, as in the case where  $P_i$  is honest  $A_i$  does not see anything from the interaction):

1. When  $\text{Sim}_i$  receives from  $\mathcal{Z}$  a request to invoke  $\mathcal{F}_{\text{inp}}$  with input  $x'$ , if a request of this type has already been received he ignores it. Otherwise: he sends  $x'$  to the functionality  $\mathcal{F}$ , who acknowledges the reception. Subsequently,  $\text{Sim}_i$  choose some random value

$r'_i$ , and computes decommitment information  $\text{Dec}(x'_i)$  and  $\text{Dec}(r'_i)$ ; furthermore,  $\text{Sim}_i$  computes commitments  $\text{Com}(1)$  and  $\text{Com}(\perp)$ . Finally,  $\text{Sim}_i$  hands  $(y', \vec{C})$  to  $\mathcal{Z}$ , where  $\vec{C} = (\text{Dec}(x'_i), \text{Dec}(r'_i), \text{Com}(1), \text{Com}(\perp))$ .

2. When  $\text{Sim}_i$  receives from  $\mathcal{Z}$  a request to invoke  $\mathcal{F}_{\text{comp}}$  with input  $x'$ : he notifies  $\mathcal{F}$ , which acknowledges the activation (to ensure output synchronization) and answers back to  $\text{Sim}_i$  with either  $y$  or  $\perp$ .  $\text{Sim}_i$  sets  $y'$  to this answer.  $\text{Sim}_i$  computes (fresh) decommitment information  $\text{Dec}(x'_i)$  and  $\text{Dec}(r)$ ; furthermore,  $\text{Sim}_i$  computes (fresh) commitments  $\text{Com}(1)$  and  $\text{Com}(\perp)$ . Finally,  $\text{Sim}_i$  hands  $(y', \vec{C})$  to  $\mathcal{Z}$ , where  $\vec{C} = (\text{Dec}(x'_i), \text{Dec}(r'_i), \text{Com}(1), \text{Com}(\perp))$ .

We argue that the simulators described above produce a view, for  $\mathcal{Z}$ , which is indistinguishable from the view of the execution of  $\pi_{\mathcal{F}_{\text{inp}}^*, \mathcal{F}_{\text{comp}}^*, \mathcal{F}_{\text{ideal}}^*}$ . One of the difficulties is the fact that at round  $\text{rmax}$  the output  $y'$  is set to the actual output of the computation. However, this behavior is simulated with the help of the synchronizing-shell, which ensures that also the simulators will be informed (and given the output) as soon as the simulated protocol reaches round  $\text{rmax}$ . The indistinguishability of the real and the simulated view follows then from the GUC security of the used commitment scheme. Indeed, the only difference in the real/simulated view is that in the real view  $\mathcal{Z}$  sees commitments on the actual round  $\rho$  and state  $M_i$  of the protocol (or to  $\rho$  and  $\text{ABRT}$ 's in case it forces an early abort), whereas in the simulated execution those are commitments to the default values  $\rho = 1$  and  $M_i = \perp$ . The GUC security of the commitment scheme ensures that the environment cannot distinguish between these two sequences of commitments.  $\square$

**Sub-step 3.2.** Second, we observe that, by definition,  $\pi_{\text{inp}}$  and  $\pi_{\text{comp}}$  are CP secure realizations of functionalities  $\mathcal{F}_{\text{inp}}$  and  $\mathcal{F}_{\text{comp}}$ , respectively. The composition theorem applied to Claim 5.4 implies that  $\pi^*$  CP-securely realizes the functionality  $\mathcal{F}$ .

**Step 4.** We next show the GUC fallback, i.e., that when  $\text{P}_M = \text{P}_{M^*}$  then the protocol  $\pi^*$  CP-securely realizes the functionality  $\{\mathcal{F}, \mathcal{R}_{\text{ins}}\}$ , where  $\mathcal{R}_{\text{ins}}$  denotes a complete network of insecure channels among the parties in  $[n]$ . This is also split in two sub-steps, along the lines of Step 3:

**Sub-step 4.1.** As in Step 3.1, we start with the  $\{\mathcal{F}_{\text{inp}}^*, \mathcal{F}_{\text{comp}}^*, \mathcal{F}_{\text{ideal}}^*\}$ -hybrid protocol  $\pi_{\mathcal{F}_{\text{inp}}^*, \mathcal{F}_{\text{comp}}^*, \mathcal{F}_{\text{ideal}}^*}^*$  (note however that now  $\text{P}_M$  executes the code defined by  $\text{P}_{M^*}$ ).

**Claim 5.5.** *Assuming  $\text{P}_M = \text{P}_{M^*}$  is honest, the protocol  $\pi_{\mathcal{F}_{\text{inp}}^*, \mathcal{F}_{\text{comp}}^*, \mathcal{F}_{\text{ideal}}^*}^*$  CP-securely realizes the functionality  $\{\mathcal{F}, \mathcal{R}_{\text{ins}}\}$ .*

*Proof.* By assumption of Lemma 5.3 we know that  $\{\bar{\mathcal{G}}, \mathcal{R}_{\text{ins}}, \mathcal{F}\} \sqsubseteq_{\pi}^{CP} \{\bar{\mathcal{G}}, \mathcal{R}_{\text{ins}}\}$ . In order to show the statement of the claim, it suffices to show that  $\pi_{\mathcal{F}_{\text{inp}}^*, \mathcal{F}_{\text{comp}}^*, \mathcal{F}_{\text{ideal}}^*}^*$  CP emulates the  $\mathcal{R}_{\text{ins}}$ -hybrid protocol  $\pi$ .<sup>29</sup> Indeed, this would imply that

$$\{\bar{\mathcal{G}}, \mathcal{R}_{\text{ins}}, \mathcal{F}\} \sqsubseteq_{\pi_{\mathcal{F}_{\text{comp}}^*, \mathcal{F}_{\text{inp}}^*, \mathcal{F}_{\text{ideal}}^*}^*}^{CP} \{\bar{\mathcal{G}}, \mathcal{R}_{\text{ins}}, \mathcal{F}_{\text{comp}}^*, \mathcal{F}_{\text{inp}}^*, \mathcal{F}_{\text{ideal}}^*\} \quad (6)$$

hence, in order to obtain the statement of the claim from Reduction 6 we need to remove  $\mathcal{R}_{\text{ins}}$  from the left-hand side. To this direction, we observe that

$$\{\bar{\mathcal{G}}, \mathcal{R}_{\text{ins}}\} \sqsubseteq_{\mathcal{D}_{\mathcal{F}_{\text{ideal}}^*}}^{CP} \{\bar{\mathcal{G}}, \mathcal{F}_{\text{ideal}}^*\}, \quad (7)$$

where  $\mathcal{D}_{\mathcal{F}_{\text{ideal}}^*}$  denotes the dummy  $\mathcal{F}_{\text{ideal}}^*$ -hybrid protocol. Indeed, similar to the dummy-mediator lemma, in order to simulate the behavior of  $\mathcal{R}_{\text{ins}}$  in the  $\mathcal{F}_{\text{ideal}}^*$ -hybrid world, the simulators use the

<sup>29</sup>Recall that  $\pi$  is an  $n$ -party protocol which includes no instruction from  $\text{P}_M$ .

party  $P_{M^*}$  that allows them to communicate arbitrarily. Using Reduction 7 and the composition theorem, we can remove  $\mathcal{R}_{\text{ins}}$  from the right side of Reduction 6.

In the remaining of the proof we show that  $\pi_{\mathcal{F}_{\text{inp}}^*, \mathcal{F}_{\text{comp}}^*, \mathcal{F}_{\text{ideal}}^*}$  CP emulates protocol  $\pi$ . To this direction, we considered a modified version of the protocol  $\pi_{\mathcal{F}_{\text{comp}}^*, \mathcal{F}_{\text{inp}}^*, \mathcal{F}_{\text{ideal}}^*}$  as follows: the hybrid functionalities  $\mathcal{F}_{\text{comp}}$  and  $\mathcal{F}_{\text{inp}}$  are removed and each honest  $P_i$  executes the code of these functionalities internally and sends to  $P_M$  his outputs over the channel  $\mathcal{F}_{\text{ideal}}(P_i, P_M)$ . More concretely, any call to  $\mathcal{F}_{\text{inp}}(P_M, P_i)$  or  $\mathcal{F}_{\text{comp}}(P_M, P_i)$  (in protocol  $\pi_{\mathcal{F}_{\text{comp}}^*, \mathcal{F}_{\text{inp}}^*, \mathcal{F}_{\text{ideal}}^*}$ ) is replaced by  $P_M$  handing his input (for  $\mathcal{F}_{\text{inp}}$  or  $\mathcal{F}_{\text{comp}}$ ) to  $P_i$ , who executes the code of the corresponding functionality and sends  $P_M$  his output (i.e., his share of the state and the next round messages) and to the environment the output that he (i.e.,  $P_i$ ) would receive from the functionality. We denote this modified protocol as  $\pi^{*'}$ . We show that:

- i. the protocol  $\pi_{\mathcal{F}_{\text{comp}}^*, \mathcal{F}_{\text{inp}}^*, \mathcal{F}_{\text{ideal}}^*}$  CP-emulates the protocol  $\pi^{*'}$
- ii. the protocol  $\pi^{*'}$  CP-emulates the protocol  $\pi$

and then use the composition theorem to conclude that  $\pi_{\mathcal{F}_{\text{comp}}^*, \mathcal{F}_{\text{inp}}^*, \mathcal{F}_{\text{ideal}}^*}$  CP-emulates the protocol  $\pi$ .

We start with Step i: We argue that protocol  $\pi_{\mathcal{F}_{\text{comp}}^*, \mathcal{F}_{\text{inp}}^*, \mathcal{F}_{\text{ideal}}^*}$  SCP-emulates the protocol  $\pi^{*'}$  (from this we can directly derive that it also CP emulates it by means of Theorem 2.3): We describe a simulator-profile  $\text{Sim}_{[n]}$  for the dummy adversary-profile  $A_{[n]}$  (denote by  $\mathcal{I} \subseteq [n]$  the set of corrupted players):

1. For  $i \in ([n] \cup \{P_M\}) \setminus (\mathcal{I} \cup \{1\})$   $\text{Sim}_i$  simply follows  $\mathcal{Z}$ 's instructions as the dummy adversary would do. This is a sound simulation because both the adversary and the simulator of such an honest party is completely cut-off by the assumed hybrids).
2. Let us consider the adversary  $A_i$  of some corrupted  $i \in \mathcal{I}$ : other than performing local computation,  $\mathcal{Z}$  might give  $A_i$  the instruction to send a message to  $P_{M^*}$  over the ideal channel. When the simulator  $\text{Sim}_i$  receives such an instruction, he simply sends this message to  $P_{M^*}$ . Any message which  $\text{Sim}_i$  receives from  $P_{M^*}$ , he forwards it to  $\mathcal{Z}$ . Additionally,  $\text{Sim}_1$  might receive from  $\mathcal{Z}$  messages of the form  $(\text{send}, m, ID)$  to be sent to  $P_{M^*}$ ; when  $\text{Sim}_1$  receives such a message he simply forwards it to  $P_{M^*}$ .

It is straight-forward to verify that with the above simulator protocol  $\pi_{\mathcal{F}_{\text{comp}}^*, \mathcal{F}_{\text{inp}}^*, \mathcal{F}_{\text{ideal}}^*}$  SCP-emulates the protocol  $\pi^{*'}$ .

We go on with Step ii: The idea of the simulation is the following: the distribution of  $\pi$ -messages exchanged by parties executing  $\pi^{*'}$  is identical to the input/output distribution of parties executing  $\pi$ . Hence the simulators needs only to simulated the commitments which  $P_{M^*}$  receives as part of the shared state between  $P_{M^*}$  and honest parties  $P_i$ .

Because the simulators can use the complete network of insecure channel to coordinate their actions, we assume that the only simulator who has a non-trivial program is  $\text{Sim}_1$ . Every  $\text{Sim}_i$  with  $i \neq 1$  acts as “slave” of  $\text{Sim}_1$ , i.e. forwards everything he receives to  $\text{Sim}_1$  and follows  $\text{Sim}_1$ 's instructions. The simulator  $\text{Sim}_1$  works as follows:

1. He makes sure, by using  $\mathcal{R}_{\text{ins}}$  to co-ordinate with the slave-simulators, that whenever  $\mathcal{Z}$  expect to see some message from some adversary  $A_i$ , it is  $\text{Sim}_i$  who outputs the corresponding simulated message to  $\mathcal{Z}$ .

2. He keeps track of the current round index  $\rho$ .  $\text{Sim}_1$  can do that as he has the overview of the execution of  $\pi$ , i.e., he gets informed for every activation coming from  $\mathcal{Z}$ .
3. For each round  $\rho$  (initially  $\rho = 0$ ) and each honest party  $P_i$ ,  $\text{Sim}_1$  stores in a variable  $M'_i$  the message-bursts  $\vec{m} \in (\mathcal{M} \cup \{\perp\})^{n-1}$  that  $P_i$  receives in the execution of protocol  $\pi$  ( $M'_i$  is updated similar to how  $\mathcal{F}_{\text{inp}}$  and  $\mathcal{F}_{\text{comp}}$  update  $M_i$ ). As in the original protocol,  $M'_i$  is an  $\text{rmax} \times (n - 1)$  matrix initially filled with  $\perp$ 's.
4. Whenever in the simulated  $\pi^{*'} P_{M^*}$  would interact with some honest party  $P_i$  to have him run the code of  $\pi_{\text{inp}}$ ,  $\text{Sim}_1$  simulates  $P_{M^*}$ 's output, i.e.,  $P_{M^*}$ 's share  $\vec{D} = [\text{Com}(x_i), \text{Com}(r_i), \text{Dec}(0), \text{Dec}(M_i)]$ , by  $\vec{D}' = [\text{Com}(0), \text{Com}(r'_i), \text{Dec}(0), \text{Dec}(M'_i)]$  for some uniformly random value  $r'_i$ . Finally,  $\text{Sim}_1$  sets  $P_{M^*}$ 's output to  $(\vec{out}', \vec{D}')$ .
5. Whenever in the simulated  $\pi^{*'} P_{M^*}$  would interact with some honest party  $P_i$  to have him run the code of  $\pi_{\text{comp}}$ ,  $\text{Sim}_1$  simulates  $P_{M^*}$ 's output, i.e.,  $P_{M^*}$ 's share  $\vec{D} = [\text{Com}(x_i), \text{Com}(r_i), \text{Dec}(\rho), \text{Dec}(\perp)]$  and the message-burst  $\vec{out}$  for the current round as follows:  $\text{Sim}_1$  hands  $P_i$  his round  $\rho$  messages (which are copied from  $M'_i$ ) and receives back the messages for round  $r + 1$ .  $\text{Sim}_1$  sets  $\vec{out}'$  to be the messages he received from  $P_i$  and uses these messages to update the corresponding entries in  $M'_i$ . Subsequently, he simulates the vector  $\vec{D} = [\text{Com}(x_i), \text{Com}(r_i), \text{Dec}(\rho), \text{Dec}(M_i)]$  by  $\vec{D}' = [\text{Com}(0), \text{Com}(r'_i), \text{Dec}(\rho), \text{Dec}(M'_i)]$ . Finally,  $\text{Sim}_1$  sets  $P_{M^*}$ 's output to  $(\vec{out}', \vec{D}')$ .

It is straight forward to verify that for every honest  $P_i$  the simulated  $M'_i$  is distributed identically to  $M_i$  that  $P_i$  would store in the execution of  $\pi^{*'}$ . Indeed, the functionalities  $\mathcal{F}_{\text{inp}}$  and  $\mathcal{F}_{\text{comp}}$  correspond to the parties' state-transition functions, and are defined to generate the same protocol-messages as the party would generate if they would be executing the protocol  $\pi$ . Hence the only difference between the real and the simulated transcript is that for each honest  $P_i$ , the commitments  $\text{Com}(x_i)$  and  $\text{Com}(r_i)$  to  $P_i$ 's input  $x_i$  and randomness  $r_i$  are replaced by commitments  $\text{Com}(0)$  and  $\text{Com}(r'_i)$ . The GUC-security of our commitment scheme ensures that  $\mathcal{Z}$  cannot distinguish between the two views.  $\square$

**Sub-step 4.2.** As in Sub-step 3.2, we observe that, by definition,  $\pi_{\text{inp}}$  and  $\pi_{\text{comp}}$  are CP secure realizations of functionalities  $\mathcal{F}_{\text{inp}}$  and  $\mathcal{F}_{\text{comp}}$ , respectively. The composition theorem applied to Claim 5.4 implies that  $\pi^*$  CP-securely realizes the functionality  $\mathcal{F}$ .

This completes the proof of Lemma 5.3.

### 5.3 A Concrete Instance

Thus far the results have been stated for an abstract GUC-AuthComplete setup. For a concrete instance we can use the KRK setup of [11, 16]. Recall that in those works the Key Registration with Knowledge (KRK) functionality (hence forth the GUC-KRK) allows the (monolithic) adversary to register and/or ask for the key of any corrupt player. We modify this to obtain the CP setup KRK such that the  $i^{\text{th}}$  adversary is allowed only to register and ask for the keys of the  $i^{\text{th}}$  player. One can easily verify that any protocol which is GUC secure in the original GUC-KRK hybrid world, is also GUC secure when using setup  $[KRK]$ .<sup>30</sup> Moreover all protocols of [11, 16] using GUC-KRK are setup off-line. In [16] the GUC-KRK is used to register public keys which allow for non-interactive

<sup>30</sup>The only difference is that when the (monolithic) adversary makes a registration or secret key request for some party to  $[KRK]$ , it needs to append the ID of this party to the message. But this has no effect on the protocol.



key agreement. Such public/secret key pairs can easily be constructed based on the Decisional Diffie-Hellman (DDH) assumption. Therefor we obtain the following corollary.

**Corollary 5.6.** *If the DDH assumption holds, there exist (efficient) setup  $\bar{\mathcal{G}}$  and a programmable resource  $\mathsf{M} = \{\mathsf{M}_x\}_{x \in \{0,1\}^*}$  such that for every well-formed aborting functionality  $\mathcal{F}$ , there exists a protocol  $\pi$  which CP realizes  $\mathcal{F}$  with GUC fallback in the  $\{\bar{\mathcal{G}}, \mathsf{M}_{\mathcal{F}}\}$ -hybrid model.*

## 6 Implications for Mechanism Design

In this section we translate our results into the language of game theory and interpret them in terms of reducing trust in mechanisms. We first translate the cryptographic language of the CP framework into that of computational game theory and define the model of games we work with. For this we adapt the language of [36] and we refer to it for a more detailed discussion. Next we formalize some relations between strategies and give the game theoretic interpretation of Theorem 5.1. Finally we describe our equivalence notion between games and use our main construction from the proof of the theorem to show how to reduce trust in the mechanism for a large class of games.

### 6.1 Viewing Protocols as Games

We view CP protocols as *mediated (extensive form) games of incomplete information* [28]; a very general model of games. We call an  $n$ -party (adversary oblivious<sup>31</sup>) functionality  $\mathcal{M}$  a *mechanism*. The environment's input  $x_i$  for player  $\mathsf{P}_i$  is its *type*, the protocol  $\pi_i$  run by  $\mathsf{P}_i$  is its *strategy* and the messages sent by  $\pi_i$  are called *actions*. We call  $\mathsf{P}_i$ 's view at the end of an execution its *output*. The view includes  $x_i$ , the random tape, a description of  $\pi_i$  and all messages received during the execution.

We call a vector of these objects with one dimension per user *profiles*. We use standard notation to construct strategy profiles as  $(\pi_{-i}, \phi_i) = (\phi_1, \dots, \phi_{i-1}, \pi_i, \phi_{i+1}, \dots, \phi_n)$ .

A game is also equipped with an efficiently computable *interpretation* function  $\Omega$  mapping output profiles to *outcome* profiles in an arbitrary outcome space.<sup>32</sup> Moreover, for each player  $\mathsf{P}_i$  we assume the existence of a real valued functions  $\mu_i$  called the *utility function* defined over the outcome space. For a outcome  $y$  the value of  $\mu_i(y) \in \mathbb{R}$  is denoted as the *payoff* of  $\mathsf{P}_i$ .

In a mediated game  $\Gamma$  players are perfectly isolated from each other beyond their interfaces with the mechanism  $\mathcal{M}$ . First they are given their types, then they run their strategy with  $\mathcal{M}$  eventually obtaining an output for the execution. The game is played as a sequence of rounds  $r_1, r_2, \dots$ . In each round  $r_\ell$  all players take an action. Once all players are done  $\mathcal{M}$  computes a vector of messages  $m_\ell$  (which may depend on all previous actions taken in the game) and sends player  $\mathsf{P}_i$  component  $m_{\ell,i}$ . The notion of *rationality* predicts that each player chooses their strategies so as to maximize the expected value of their payoff.

**Definition 6.1** (Mediated Game). *A computational extensive form mediated game of incomplete information (or just game for short) is a 4-tuple  $\Gamma = (\mathcal{M}, \Omega, \mu, T)$  with mechanism  $\mathcal{M}$ , interpretation function  $\Omega$ , utility profile  $\mu$ , and public distribution over type profiles  $T$ . Additionally  $\mathcal{M}, \Omega$  and  $\mu$  are efficiently computable and  $T$  is efficiently samplable.*

<sup>31</sup>The output of an adversary oblivious functionality is defined as not depending on the adversarial interfaces.

<sup>32</sup>Keeping in line with standard game theoretic models we restrict the output of  $\Omega$  to not depend on the choice of strategy used (i.e. the value of the code for the ITM in the views).

## 6.2 CP Realizing Mechanisms

We begin by interpreting the definition of CP realization in the game theoretic setting. For this we develop a formal means for comparing and relating games to each other.

Using “small o-notation” we denote by  $\text{NEGL} = \{v(\cdot) \mid \forall c \in \mathbb{N}, v \in o(\lambda^{-c})\}$  the set of negligible functions in security parameter  $\lambda$ . For game  $\Gamma = (\mathcal{M}, \Omega, \mu, T)$  let  $S(\Gamma)$  be the set strategy profiles. For  $\pi \in S(\Gamma)$  let  $\mu_i(\pi)$  be a random variable over  $\mathbb{R}$  describing  $P_i$ 's payoff induced by playing  $\Gamma$  with  $\pi$  using type profile  $t \leftarrow T$  and fresh randomness for  $\pi$  and  $\mathcal{M}$  and let  $\bar{\mu}_i(\pi) = \mathbb{E}(\mu_i(\pi))$  be it's the expected value. A pair of strategy profiles are equivalent if they induce the same payoff profiles. That is for games  $\Gamma$  and  $G$  with utility profiles  $\mu$  and  $\nu$  we call strategy profile  $\pi \in S(\Gamma)$  and  $\phi \in S(G)$  *equivalent* (write  $\mu(\pi) \approx \nu(\phi)$ ) if:

$$\forall D \in \text{PPT} \quad \Pr[p_0 \leftarrow \mu(\pi), p_1 \leftarrow \nu(\phi), b \leftarrow \{0, 1\} : D(p_b) = 1] \in \text{NEGL}.$$

In this case we abuse notation and write  $\pi \approx \phi$ .<sup>33</sup>

We call a function over  $n$  dimensional vectors an *efficiently and locally computable* function if it can be described as the product of  $n$  independent efficiently computable functions where the  $i^{\text{th}}$  function maps the  $i^{\text{th}}$  input coordinate to the  $i^{\text{th}}$  output coordinate. We would like to say that a game  $\Gamma$  “contains” a game  $G$  if for every strategy profile in  $G$  there is an equivalent strategy profile in  $\Gamma$ . As we work in a computational setting though we make the additional requirement that the mapping be efficiently and locally computable.

**Definition 6.2** (Containing a Game). *Let  $\Gamma = (\mathcal{M}, \Omega, \mu, T)$  and  $G = (\mathcal{R}, \Pi, \mu, T)$  be a pair of  $n$ -party mediated games. We say that  $\Gamma$  contains  $G$  if there exists an efficiently and locally computable function  $f : S(G) \rightarrow S(\Gamma)$  such that:*

$$\forall \phi \in S(G) \quad \pi \approx f(\pi).$$

*In this case we write  $G \subseteq \Gamma$ .*

We are now ready to prove the primary tool which relates the CP framework to that of computational games. Intuitively, the following theorem states that if a mechanism  $\mathcal{R}$  can be used to CP realize a mechanism  $\mathcal{M}$ , then for any game  $\Gamma$  using  $\mathcal{M}$  there exists a game  $G$  using  $\mathcal{R}$  (with a different interpretation function but) with the same types and utilities with  $G \subseteq \Gamma$ .

**Theorem 6.3.** *Let  $\mathcal{R}$  and  $\mathcal{M}$  be a pair of mechanisms such that  $\mathcal{M} \sqsubseteq^{CP} \mathcal{R}$ . Then for any computational mediated game  $\Gamma = (\mathcal{M}, \Omega, \mu, T)$  there exists a game  $G = (\mathcal{R}, \Pi, \mu, T)$  for which it holds that  $G \subseteq \Gamma$ .*

*Proof.* We construct a game  $G$  and prove the theorem. As  $\mathcal{R}$ ,  $\mu$  and  $T$  are fixed it only remains to define interpretation function  $\Pi$  and utility profile  $\nu$ .

Let  $\Upsilon$  be a function which maps  $\mathcal{R}$ -hybrid views to computationally indistinguishable  $\mathcal{M}$ -hybrid views. More precisely  $\Upsilon$  takes as input an  $n$ -tuple of views (one per player) generated by an execution in the  $\mathcal{R}$ -hybrid world using arbitrary (efficient)  $n$ -party protocol  $\phi_{[n]}$  in the  $\mathcal{F}$ -hybrid world.

1. It extracts the description of  $\phi_{[n]}$ , their inputs  $x_{[n]}$  and all random tapes used to run  $\phi_{[n]}$ .
2. It computes a description of the protocol  $\sigma_{[n]} = \text{Sim}_{[n]}(\phi_{[n]})$ .

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<sup>33</sup>We observe that this is stronger then only comparing the expected values of the two distributions; the only property of payoffs considered in an computational Nash equilibria. In fact any poly-time testable property of the payoff distributions is preserved in the above equivalence notion.

3. It selects fresh randomness for  $\mathcal{M}$  and runs  $\sigma$  on input  $x_{[n]}$  with the matching extracted random tapes to simulating an execution in the  $\mathcal{M}$ -hybrid world.
4. Finally it outputs the  $n$ -tuple of views of  $\phi_{[n]}$  generated by the simulation.

Then let  $\Pi = \Upsilon \circ \Omega$  be the function that takes as input an  $n$ -tuple of views of a  $\mathcal{R}$ -hybrid execution and maps them to a real number by first applying  $\Upsilon$  and then  $\Omega$ .

It remains to prove that  $G$  is contained in  $\Gamma$ . Assume for the sake of contradiction that  $G \not\subseteq \Gamma$ . That is  $\exists \phi \in S(G)$  such that for any efficiently and locally computable function  $f : S(G) \rightarrow S(\Gamma)$  there exists an efficient distinguisher  $D$  for  $\nu(\phi)$  and  $\mu(f(\phi))$ . We use  $\phi$  and  $D$  to construct adversaries and an environment for which there are no simulators contradicting  $\mathcal{M} \sqsubseteq^{CP} \mathcal{R}$ .

Let  $A_{[n]}$  be the set of  $\mathcal{R}$ -hybrid world adversaries which corrupt their respective players and run  $\phi$  on them with input (type) supplied by the environment  $\mathcal{Z}$  and at the end of the execution return their entire view to  $\mathcal{Z}$ . By definition a mechanism is an adversary oblivious functionality and so it ignores adversarial interfaces. In other words the actions of any adversaries  $\sigma_{[n]}$  in the  $\mathcal{M}$ -hybrid world depend only on their input from  $\mathcal{Z}$ , random tapes and the messages they receive on the corrupt players interface to  $\mathcal{M}$ . In particular they can be viewed as a strategy profile  $\pi \in S(\Gamma)$  where the  $i^{\text{th}}$  strategy  $\pi_i$  behaves precisely as  $\sigma_i$  simply dropping any output  $\sigma_i$  might produce on it's adversarial interface to  $\mathcal{M}$  and letting player  $P_i$  act on behalf of the environment (in order to supply a type argument).

Consider the environment  $\mathcal{Z}_\sigma$  which samples  $t \leftarrow T$ , gives it to  $A_{[n]}$  and then runs  $D$  on the  $n$ -tuple of views it receives as output. Then by assumption of  $D$  distinguishes between the outputs of  $\sigma_{[n]}$  and  $A_{[n]}$ . But this implies that for every set of  $\mathcal{M}$ -hybrid simulators  $\sigma_{[n]}$  for  $A_{[n]}$  there exists a distinguishing  $\mathcal{Z}_\sigma$  which contradicts  $\mathcal{M} \sqsubseteq^{CP} \mathcal{R}$ .  $\square$

### 6.3 Reducing Trust in Mechanisms

In this subsection we prove Theorem 6.8. For this we first formalize what it means for a pair of games to be equivalent. Then we prove a lemma stating that if two games contain each other then they are equivalent. Next we prove a lemma that, for any game with mechanism  $\mathcal{M}$  constructs an equivalent game but with the synchronization wrapper  $\widehat{\mathcal{M}}$  applied to the mechanism. Finally we prove the central theorem of this section showing how to reduce the trust in a mechanism while building an equivalent game.

#### 6.3.1 Equivalent Games

The following definition of a computational Nash Equilibrium is taken from [37]. We note that these are one of the most general types of (computational) equilibria and so if we can preserve these between games then in particular we also preserve other more resilient (and desirable) equilibria such as  $k$ -resilient equilibria, dominant strategies and also correlated equilibria.

**Definition 6.4** (Computational NE). *For game  $\Gamma = (\mathcal{M}, \Omega, \mu, T)$  strategy profile  $\pi \in S(\Gamma)$  is called a computational Nash Equilibrium (cNE) if:*

$$\forall \phi \in S(\Gamma) \quad \exists \epsilon \in \text{NEGL} \quad \forall i \in [n] : \bar{\mu}_i(\pi) \geq \bar{\mu}_i(\phi_i, \pi_{-i}) - \epsilon.$$

*We denote the set of cNE of  $\Gamma$  with  $E(\Gamma) = \{\pi \in S(\Gamma) \mid \pi \text{ is a cNE}\}$ .*

As before, besides equating the sets of cNE we make the additional requirement that there be an efficiently and locally computable function mapping cNE in one game to cNE in the other.

**Definition 6.5** (Equivalent Computational Games). *We call a pair of  $n$ -party computational mediated games  $\Gamma = (\mathcal{M}, \Omega, \mu, T)$  and  $G = (\mathcal{R}, \Pi, \mu, T)$  equivalent if there exist a pair of efficiently and locally computable functions  $f : E(\Gamma) \rightarrow E(G)$  and  $g : E(G) \rightarrow E(\Gamma)$  such that:*

- $\forall \pi \in E(\Gamma) \quad \pi \approx f(\pi)$
- $\forall \phi \in E(G) \quad \phi \approx g(\phi)$ .

In this case we abuse notation and write  $\Gamma \approx G$ .

It is easy to verify that  $\approx$  is transitive. Next we show that if a pair of games contain each other then they are equivalent.

**Lemma 6.6.** *For  $n$ -party mediated games  $\Gamma$  and  $G$ :*

$$\Gamma \subseteq G \quad \wedge \quad G \subseteq \Gamma \quad \implies \quad G \approx \Gamma.$$

*Proof.* We need to show that every cNE in  $\Gamma$  can be efficiently and locally mapped to cNE in  $G$  and vice versa. We show the first part and the second follows with an identical argument switching the roles of  $G$  and  $\Gamma$ .

Let  $f : S(\Gamma) \rightarrow S(G)$  be the mapping given by  $\Gamma \subseteq G$ . Let equilibria  $\pi \in E(\Gamma)$  with  $g(\pi) = \phi \in S(G)$ . Suppose for the sake of contradiction that  $\phi \notin E(G)$ . This implies that for some  $i \in [n]$  there is a strategy  $\phi'_i$  with an expected payoff significantly<sup>34</sup> larger than that of  $\phi$ . In symbols  $\bar{v}(\phi) \leq \bar{v}(\phi'_i, \phi_{-i}) + \delta$  for some function  $\delta$  growing faster than any function in NEGL. By Definition 6.2  $f$  is locally computable and preserves payoff profiles it follows that there exists a strategy  $\pi'_i$  for  $P_i$  in  $\Gamma$  such that:

$$\bar{\mu}(\pi) \approx \bar{\mu}(f(\phi)) \approx \bar{\mu}(\phi) \leq \bar{\mu}(\phi'_i, \phi_{-i}) + \delta \approx \bar{\mu}(f(\phi'_i, \phi_{-i})) + \delta \approx \bar{\mu}(\pi'_i, \pi_{-i}) + \delta$$

which contradicts  $\pi \in E(\Gamma)$ . □

### 6.3.2 On Output Synchronization in a Stand-Alone Game

To formally apply the results of Theorem 5.1 we first need to address the issue of output synchronization. As the CP security notion is composable it is equipped with an online distinguisher (namely environment  $\mathcal{Z}$ ). Thus a protocol which takes say 2 rounds of communication to produce output can at most realize a functionality which waits till all players have been activated 2 times (in an appropriate order) before it gives them output. Otherwise  $\mathcal{Z}$  can trivially distinguish by activating all player once and then checking if they already have output before it continues the execution. (See Section 4 for a more detailed discussion.)

While we briefly address the more general composable mediated games setting later on, for now we take the common view in game theory and consider a game as being an isolated event (or at least that no players are involved in concurrent interactions). In this setting output synchronization has no effect on the game.

To see why this is we first describe the effects of the output synchronization wrapper in game theoretic terms. Let  $\mathcal{M}$  be an efficient mechanism and  $\widehat{\mathcal{M}}$  be it's output synchronized version such that we are given a  $\mathcal{R}$ -hybrid protocol  $\pi$  which CP realizes it. Moreover let  $u$  be an upper-bound on the number of rounds of  $\pi$ . As we need  $\widehat{\mathcal{M}}$  to be adversary oblivious we fix the value of  $R = u$  in it's wrapper instead of having it query an simulator for the value. This guarantees that on the one hand simulators in the  $\widehat{\mathcal{M}}$ -hybrid world obtain at least as much output synchronization as obtained from an execution of  $\pi$ , but on the other hand  $\widehat{\mathcal{M}}$  is adversary oblivious making it a valid mechanism.

Now when viewed as mechanisms  $\widehat{\mathcal{M}}$  behaves exactly as  $\mathcal{M}$  except that for any round  $r_\ell$  the mechanism only produces the output  $m_\ell$  once all players have made (at least)  $R$  attempts at taking

<sup>34</sup>By "significant" we mean a function in  $\lambda$  larger than any  $\epsilon \in \text{NEGL}$ .

action for  $r_\ell$ . Although the first attempted action is the one that is actually used to compute  $m_\ell$ , mechanism  $\widehat{\mathcal{M}}$  does not deliver  $m_\ell$  until all players have made  $R$  attempts.

Translating this observation to the (stand-alone) game theoretic setting we obtain the following lemma.

**Lemma 6.7.** *For any mechanism  $\mathcal{M}$  and  $\widehat{\mathcal{M}}$  and computational mediated game  $\Gamma = (\mathcal{M}, \Omega, \mu, T)$  there exists an interpretation function  $\Pi$  such that the computational mediated game  $G = (\widehat{\mathcal{M}}, \Pi, \mu, T)$  is equivalent to  $\Gamma$ .*

*Proof.* We show the lemma in two parts. First we show that  $G \subseteq \Gamma$  and then we show that  $\Gamma \subseteq G$  which implies  $\Gamma \approx G$  by Lemma 6.6.

Fix any positive integer  $R$  (of size polynomial in the security parameter). To show  $G \subseteq \Gamma$  we first define an efficient and locally computation function  $g : S(G) \rightarrow S(\Gamma)$ . It consists of  $n$  efficient functions  $g_i$  that take input a strategy  $\phi_i$  and output strategy  $\phi'_i$ . The ITM  $\phi'_i$  runs  $\phi_i$  internally such that for all rounds  $r_\ell$  once  $\phi_i$  has made its first attempt taking action, any resulting messages  $m_{\ell,i}$  from  $\mathcal{M}$  to  $\phi_i$  are only made visible to  $\phi_i$  once it has made (at least)  $R - 1$  additional attempts taking action. The remaining  $R - 1$  attempts are simply dropped. Other than that  $\phi'_i$  behaves exactly as directed by it's internal copy of  $\phi_i$ .

We are now ready to construct interpretation function  $\Pi$  from  $\Omega$ . Function  $\Pi$  takes in an  $n$ -tuple of outcomes of a run of  $G$ . Much like the function  $\Upsilon$  described in the proof of Theorem 6.3  $\Pi$  extracts the strategy profile  $\phi$ , the types and the random tapes used in  $G$  to runs  $g(\phi)$  with the same inputs in a new game with mechanism  $\mathcal{M}$ . Thus by construction, for every strategy profile  $\phi \in S(G)$  there exists strategy profile  $g(\phi) \in \Gamma$  inducing identical payoff distribution. In particular  $G \subseteq \Gamma$ .

Next we show that  $\Gamma \subseteq G$ . We define an efficient and locally computable function  $f : S(\Gamma) \rightarrow S(G)$ . It too consists of  $n$  efficient functions  $f_i$  that take in a strategy  $\pi_i$  for  $\Gamma$  and output a strategy  $\pi'_i$  for  $G$ . The only difference between  $\pi_i$  and  $\pi'_i$  is that for every action taken by the former the later repeats the action an additional  $R - 1$  times. Then by construction of  $f$  (i.e.  $\Pi$ ) and the fact that  $\Omega$  does not depend on the particular choice of strategy used to compute the output profile we get:

$$\Omega(\pi) = \Omega(g(f(\pi))) = \Pi(f(\pi))$$

which implies that  $\Gamma \subseteq G$ . □

### 6.3.3 Implications of CP Compiler to Game Theory

We apply the game theoretic interpretation of CP security (Theorem 6.3) to our main feasibility result for CP security with GUC fallback (Theorem 5.1). This results in a transformation mapping any mediated game  $\Gamma = (\mathcal{M}, \Omega, \mu, T)$  to an equivalent mediated games  $G = (\mathcal{R}, \Pi, \mu, T)$  which places significantly less trust it's mechanisms. Borrowing some cryptographic concepts the change in trust intuitively be expressed as follows: while  $\Gamma$  relies on  $\mathcal{M}$  to act as an ideal functionality computing the output correct, the new game  $G$  only trusts  $\mathcal{R}$  to act as a network of insecure channels over which a GUC secure protocol is run.<sup>35</sup> In particular players in  $G$  no longer have to trust the mechanism to maintain the privacy of their actions nor do they have to trust that  $\mathcal{R}$  computes the correct output. On the other hand they do still have to trust the mechanism to enforce isolation of colluding players as well as fairness.

<sup>35</sup>A more formal interpretation is obtained from the fallback security achieved in Theorem 5.1.

**Well Motivated Games.** We view any game as having a special input  $\perp$  modeling the case when a player aborts the game or simply refuses to play. The construction of Theorem 5.1 permits simulating a mechanism which handles such actions by producing outcome  $\perp$  for all players. However we opt instead to place the reasonable assumption on the utility profile of the game ruling out such behavior as irrational. In particular we call a game  $\Gamma$  *well motivated* if for all  $i \in [n]$  the expected utility of any outcome obtained with an abort is less than the utility of all outcomes obtained without an abort.

Next we state and prove the central theorem of this section.

**Theorem 6.8** (Replacing Mechanisms). *Let  $\bar{\mathcal{G}}$  be a functionality satisfying the conditions of Theorem 5.1 and let  $\mathbf{M}$  be the corresponding programmable resource. Then for any well motivated computational mediated game  $\Gamma = (\{\mathcal{M}, \bar{\mathcal{G}}\}, \Omega, \mu, T)$  there exists a computational mediated game  $G = (\{\mathbf{M}_{\mathcal{M}}, \bar{\mathcal{G}}\}, \Pi, \mu, T)$  such that  $\Gamma \approx G$ .*

*Proof.* Using protocol compiler in the proof of Theorem 5.1 we obtain a protocol  $\pi$  with number of rounds upper bounded by some fixed  $u$  such that  $\{\widehat{\mathcal{M}}, \bar{\mathcal{G}}\} \sqsubseteq_{\pi}^{CP} \{\mathbf{M}_{\mathcal{M}}, \bar{\mathcal{G}}\}$ .<sup>36</sup> Suppose we can construct a game  $G$  equivalent to  $\Gamma$  but using mechanism  $\mathcal{M}' = \{\widehat{\mathcal{M}}, \bar{\mathcal{G}}\}$  then the theorem follows directly by Lemma 6.7.

Let  $G$  be a computational mediated game with mechanism  $\mathcal{M}'$ , utility profile  $\mu$  and type profile  $T$ . We construct  $G$ 's interpretation function  $\Pi$  from  $\Omega$  as in the proof of Theorem 6.3 thus  $G \subseteq \Gamma$ . So if we can also show that  $\Gamma \subseteq G$  then the theorem follows directly from Lemma 6.6.

To show that  $\Gamma \subseteq G$  we define a function  $g : S(\Gamma) \rightarrow S(G)$  and show that it satisfies Definition 6.2. By assumption there exists a protocol (strategy profile)  $\phi$  such  $\mathcal{M} \sqsubseteq_{\phi}^{CP} \mathcal{R}$ . Let  $\pi \in S(\Gamma)$ . Then define  $g(\pi) := \pi \circ \phi$  which is the protocol that uses  $\phi$  in the  $\mathcal{R}$ -hybrid world to simulate an instance of  $\mathcal{M}$  which it uses in turn to run  $\pi$ . In particular  $g$  is efficiently and locally computable as the  $i^{\text{th}}$  output component  $\pi_i \circ \phi_i$  depends only on the value of  $\pi_i$ .

Moreover for any  $\pi \in S(\Gamma)$  we claim that  $\pi \approx g(\pi)$ . Suppose for a moment that this were not so. That implies that there exists of an efficient distinguisher  $D$  for  $\mu(\pi)$  and  $\mu(g(\pi))$ . But if this were the case then an environment  $\mathcal{Z}_{\pi}$  can use  $\pi$  and  $D$  to attack  $\phi$ . More precisely  $\mathcal{Z}_{\pi}$  samples  $t \leftarrow T$  and runs  $\pi(t)$  forwarding all messages from  $\pi_i(t_i)$  for  $\mathcal{M}$  to  $P_i$  and feeding the responses back to  $\pi_i(t_i)$ . Let  $V$  be the  $n$ -tuple of views of the emulated  $\pi$ 's in the execution. Then  $\mathcal{Z}_{\pi}$  produces the output bit computed as  $D(\mu(\Omega(V)))$ . That is it interprets the views as outputs for  $\Gamma$ , computes the payoff profile and gives the result to the distinguisher returning it's output. As  $D$  is a good distinguisher so must  $\mathcal{Z}_{\pi}$  be.  $\square$

At this point a traditional game theorist will no doubt be concerned with the fact that the compiled CP protocol of Section 5 requires the use of a setup functionality. We address two issues the presence a setup may introduce.

**On Games with Setup.** As observed already in Section 6 an undesirable side effect of using our protocol compiler is the presence of a shared setup functionality.

However, we note that the setup is only used at the beginning of the computation and the interaction is independent of players type. Thus, to apply the compiler to *any* efficient mechanism (rather than ones with setup) one could also adopt a model similar to that of [31] where games are preceded by a pre-computation round which is run before types are distributed. Moreover, in contrast to [31] players need only be isolated just before they receive their types (but after they have

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<sup>36</sup>As discussed above to make  $\widehat{\mathcal{M}}$  adversary oblivious (i.e. a valid mechanism) we fix the value of  $R = u$  in the synchronization wrapper (rather than have it query the adversary).

finished the pre-computation phase). In this setting any signaling (or more generally correlation) that may occur due to the setup is not a problem as types are not known yet.

Another mitigating factor is that the CP compiler can use all known GUC setup functionalities. The KRK model of [11] calls for players to place their public keys into a publicly viewable file which they could potentially use to signal by skewing the distribution of their keys. However the Augmented CRS (ACRS) setup in the same paper only contains the master public key of an IBE. In particular the shared view of players contains a random sample from an *honestly sampled* distribution. Thus the most they can gain in this setting is added correlation (aka randomness pollution). However actually communication is no longer an option. One issue here is that [11] already assumes authenticated channels so the CP compiler requires a setup in the form of PKI to realize these (as described in [17]). But using identity based signatures [23] this too can be achieved with out having players choose values in the public view.

**Randomness Pollution and Setup.** In order to apply our CP protocol to reduce trust in mechanisms players need to interact with a setup functionality. Formally this means a game with mechanism  $\mathcal{M}$  also needs to give players access to a setup  $\bar{\mathcal{G}}$  which we denote with a mechanism of the form  $\{\mathcal{M}, \bar{\mathcal{G}}\}$  and indeed, this is the language we use in the theorem below.

However this causes *randomness pollution* i.e. the presence of unwanted randomness in the public view of players. We point out three mitigating factors. First the interaction with  $\bar{\mathcal{G}}$  can take place before types are distributed (as in [31]) after which  $\bar{\mathcal{G}}$  can disappear. Moreover if the ACRS of [11] is used in combination with identity based signatures [23] then in particular, players no longer have any influence over the public view (unlike a normal PKI such as the KRK setup where users choose their public keys). Finally the setup can be reused indefinitely across many runs of many (possibly different) games. So taking an an asymptotic view of game play (such as for repeated games) the randomness pollution tends towards 0.

## 6.4 Concurrent Games

In this section we have so far opted for the standard model of a single game being played in a stand-alone setting. However the cryptographic results obtained above would actually allow for more general statements about games running *concurrently*.

By concurrently we mean the intuitive model where each rational player may be taking part in multiple mediated games concurrently, each with different player sets and mechanisms. While mechanisms still operate only on actions of a single game, the notions of strategies, interpretations, utilities, types distributions and payoffs are generalized to cover all games a particular player is involved in simultaneously. Thus an equilibrium spans all games being run in parallel and is described as a strategy profile with one strategy for every player taking part in at least one game.

Even in this setting a CP protocol can still be used to replace a particular mechanism. However an interesting subtlety concerns the inherent synchronization a protocol might contain. Indeed, in Theorem 5.1 this is addressed by adding the synchronization shell to  $\mathcal{F}$ . We note that this issue also arises with the protocols of [28, 26, 24] as the many publicly verifiable actions also provide new means for synchronization between players. In a concurrent setting sets of players could well leverage their common view of these events in one game to coordinate actions “illegally” in other games running concurrently or even to communicate illegally.

We envisage two solutions for this issue. The first is to assume that protocols are executed atomically. Thus players can not use the more fine grained synchronization obtained from a protocol execution effectively in a different game. Indeed, on an implicit level this is the approach

of [26]. However it could be argued that such a solution does not really capture the intuition behind “universally composable” mechanism design. Instead it’s closer to sequential composability, especially for normal form games.

In order to salvage the intuition behind the nature of strong composition we propose a second variant more along the lines of Theorem 5.1. In particular extra the means for synchronization must be modeled explicitly in the original mechanism  $\mathcal{M}$ . For example suppose we can CP realize  $\mathcal{M}$  in a  $\mathcal{R}$ -hybrid world with some protocol  $\pi$  which uses  $c$  rounds. Further, suppose  $\mathcal{M}$  is modified by applying the synchronization wrapper as described above in this appendix to obtain mechanism  $\widehat{\mathcal{M}}$ . In particular at the beginning of each round  $\widehat{\mathcal{M}}$  asks for  $c$  actions from each player where only the first action per player is actually used to compute the next output message of the  $\widehat{\mathcal{M}}$ . However the output is only produced and delivered to the players once all players have made handed in their inputs  $c$ . Somewhat more precisely  $\widehat{\mathcal{M}}$  also has to ensure that the activations happen in an appropriately interleaved manner reflecting the constraints of how a protocol replacing  $\widehat{\mathcal{M}}$  would be executed. For example if  $P_i$  hasn’t made his attempt yet then any other is allowed at most one action counting towards its  $c$ .

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## A Common Elements of (G)UC Models.

In order to describe the CP framework formally first clarify the paradigms inherited from the GUC framework [11] and fix some common notation for comparing protocol executions with each other.

**Ensembles and Distance.** We briefly review the notions of distribution ensembles and (computational) indistinguishability. A pair of ensembles  $X$  and  $Y$  indexed by a set  $S$  is an infinite set of random variables each associated with a unique element in  $S$ .

**Definition A.1.** We say that a pair of distribution ensembles  $X$  and  $Y$  indexed by  $S$  are (computationally) indistinguishable if for all non-uniform PPT Turing machines  $D$  the following function  $\Delta_{X,Y}(s)$  with domain  $S$  is negligible in  $s$ :

$$\Delta_{X,Y}(s) := |\Pr[D(X_s) = 1] - \Pr[D(Y_s) = 1]|.$$

In this case we write  $X \approx Y$  to denote the relation.

**Modeling Functionalities.** As in the UC framework we model functionalities as PPT ITMs. A functionality might consist of a set of functionalities  $\mathcal{F} := \{\mathcal{F}_1, \dots, \mathcal{F}_m\}$  available to the parties in parallel, where it is assumed that there exists some addressing mechanism for the players to interact with the component functionalities  $\mathcal{F}_1, \dots, \mathcal{F}_m$ .

A primary feature of the CP model are the split adversaries. Thus the main *difference* between the CP and UC functionalities lies in the interface(s) made available to the adversaries. In addition to its interfaces to the (honest) parties an  $n$ -party resource  $\mathcal{F}$  has a dedicated interfaces to *each* of the  $n$  adversaries  $A_1, \dots, A_n$ . For any message which is sent/received on an adversarial interface, the specification of  $\mathcal{F}$  defines the adversary-ID (in particular the matching interface) of the adversary that should send/receive the message (just as is already the case for messages sent/received by honest parties).

**Simulation-based Security.** Simulation-based security definitions follow the real-world/ideal-world paradigm: In the real world, the players execute the protocol and can communicate over channels as defined by the model. In the ideal world, the players securely access an ideal functionality  $\mathcal{F}$  that obtains inputs from the players, runs the program that specifies the task to be achieved by the protocol, and returns the resulting outputs to the players. Intuitively, a protocol *securely realizes* the functionality  $\mathcal{F}$  if, for any real-world adversary  $A$  attacking the protocol execution, there is an ideal-world adversary  $\text{Sim}$ , also called the *simulator*, that emulates  $A$ 's attack in the ideal evaluation of  $\mathcal{F}$ . The “quality” of the simulation is specified by considering a distinguisher  $\mathcal{Z}$ , called the *environment*, which interacts, in a well defined manner, with the parties and the adversary or the simulator and tries to distinguish between the two worlds.

The advantage of simulation-based security is that it satisfies strong composability properties: let  $\pi_1$  be a protocol that securely realizes a functionality  $\mathcal{F}_1$ . If a protocol  $\pi_2$ , using the functionality  $\mathcal{F}_1$  as a subroutine, securely realizes a functionality  $\mathcal{F}_2$ , then the protocol  $\pi_2^{\pi_1/\mathcal{F}_1}$ , which results when replacing the calls to  $\mathcal{F}_1$  by invocations of  $\pi_1$ , securely realizes  $\mathcal{F}_2$  (without calls to  $\mathcal{R}_1$ ). therefore, it is sufficient to analyze the security of the simpler protocol  $\pi_2$  in the  $\mathcal{F}_1$ -*hybrid* model, where the players run  $\pi_2$  with access to the ideal functionality  $\mathcal{F}_1$ . For more details on composability of protocols and a formal handling of composition, the reader is referred to, e.g., [7, 18, 9, 13, 5, 35].

**UC with Global Setup (GUC).** In defining the model of execution we use as a starting point the UC with global setup (GUC) framework of Canetti et al. [11]. The main difference between UC and GUC is that in the GUC model any global setup is present in *both* the real and the ideal worlds, and is accessible by the environment. Thus, in the GUC framework, a simulator must work with the setup it is provided; this is in contrast to the UC framework where the simulator gets to *simulate* the setup (which provides the simulator with extra power). For the purpose of collusion preservation, this latter approach will not work since we will, in general, have many simulators running in parallel, with no mechanism for coordinating the setup among themselves.<sup>37</sup> It is for this reason that we use GUC as our starting point.

**Execution Notation.** In this work we often compare the outcome of executions of different arrangements of ITMs with each other. To this end we extend the notation of [9, 11] which we now briefly review as it applies to the case of the GUC framework. For more details we refer the reader to the original works. We denote by  $\bar{\mathcal{G}}$  the ACRS setup functionality of [11]. For an  $\mathcal{R}$ -hybrid protocol  $\pi$ , setup  $\bar{\mathcal{G}}$ , adversary  $A$ , and environment  $\mathcal{Z}$ , we denote the output of  $\mathcal{Z}$  after witnessing an execution of  $\pi$  in the GUC  $\mathcal{R}$ -hybrid model in the presence of  $A$  as the ensemble  $\text{EXEC}_{\pi, A, \mathcal{Z}}^{\bar{\mathcal{G}}, \mathcal{R}}$ . To be precise, by this we denote the ensemble of random variables, parameterized by the security parameter  $k$  and the auxiliary input  $z$  of the environment (for details the reader is referred to [11]). For notational simplicity, similarly to [9, 11], if for a particular player  $P_i$ , no ITM is given in the subscript then it is implicitly meant that  $P_i$  acts as the dummy player simply forwarding all messages faithfully between  $\mathcal{Z}$  and  $\mathcal{R}$ .

**Definition A.2** (UC Computation with Global Setup). *Let  $\bar{\mathcal{G}}$  be a global setup,  $\mathcal{R}$  be a resource. For an  $n$ -party efficient protocol  $\pi$  and functionality  $\mathcal{F}$  we say that  $\pi$  GUC-realizes  $\mathcal{F}$ , if  $\forall A \exists \text{Sim} \forall \mathcal{Z} :$*

$$\text{EXEC}_{\pi, A, \mathcal{Z}}^{\bar{\mathcal{G}}, \mathcal{R}} \approx \text{EXEC}_{\text{Sim}, \mathcal{Z}}^{\bar{\mathcal{G}}, \mathcal{F}}$$

## B Relations to Abstract Cryptography

We briefly discuss some of the many relations between the CP framework and the Abstract Cryptography (AC) framework of [35]. In particular we describe how our model can be described using the language of the Abstract Cryptography framework. We assume the reader is familiar with the definitions and results of [35].

We begin at the highest level of abstraction and briefly provide a concrete instance of many of the most important notions defined by the AC framework. We view CP as living at Level 3, the most concrete level of AC. In the following discussion, reflecting the absence of adversaries in AC, we use the term *party* to denote the union of the players and adversaries in CP.

**A Concrete Isomorphism.** CP uses the *pseudo-metric* of computational indistinguishability defined over sets of random variables which is, in particular, an equivalence relation. Moreover CP implicitly equips random variables with a *parallel composition* operator: informally, composed variables are simply viewed as a single variable. In the words of [35] this composition operation

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<sup>37</sup>For concreteness consider a 2-party protocol which realizes  $\mathcal{F}$ : a secure unidirectional transfer of a single bit using a CRS as setup. Suppose both parties are corrupt with the strategy that they obtain the CRS, output it and terminate. Of course in the real world their outputs will be randomly distributed and identical. However, in an ideal world with no CRS functionality but only  $\mathcal{F}$  made available to the two simulators they have no way to feed random but identical values for the CRS to their emulations.

(denoted by “ $||$ ”) is *non-expanding*. Taking this view in turn induces the CP notion of an *isomorphism*. Intuitively, in CP we consider two executions (captured by the collection of random variables they induce as the output of the environment  $\mathcal{Z}$ ) to be equivalent (i.e. isomorphic) if they are computationally indistinguishable. More precisely, the CP framework implicitly realizes the definition of isomorphisms using functions  $f$  and  $g$  that are defined via 2 sets of ITMs (including all ITMs of parties and available ideal functionalities) participating in an execution. The functions map sets of  $2n$ -inputs (one per party) to random variables describing the output of  $\mathcal{Z}$ . The relation  $\rho$  is essentially the identity relation on the possible inputs of players and the pseudo-metric ‘ $\simeq$ ’ is computational indistinguishability. One caveat is that CP implicitly only requires isomorphism to hold for input tuples which can actually be sampled by  $\mathcal{Z}$  where as AC requires isomorphism to hold for all possible inputs.

**Components and Constructors.** AC introduces the notion of a *component set* which in the case of CP consists of the set of ideal functionalities (i.e. PPT ITMs with one interface per party). As required by AC, they can be composed in parallel by making several such functionalities available to players in a hybrid world setting. As a concrete *constructor set* CP uses ITMs run by parties. Viewed this way, serial composition consists of running a “protocol stack” (as is done in several proofs in this work) while parallel composition corresponds to simply grouping the interfaces of several ITMs together into a single ITM.<sup>38</sup> Finally the neutral element of the constructor set is the dummy protocol (also called dummy adversary depending on the party).

**Choice Sets and Abstractions.** In the CP framework an *2n-choice setting* is a function defined implicitly by the specification of all ITMs taking part in an execution. It maps  $2n$ -inputs (one per party) to the output random variable of  $\mathcal{Z}$ . By extension the CP framework is primarily concerned with *2n-specifications* defined by sets of  $2n$ -choice settings that share the same ideal functionalities and protocols (but different adversaries). A statement about CP-emulation is really a statement relating two such sets. Moreover the primary instance of  *$i^{\text{th}}$  guaranteed choice domains* in a CP “real” world is the set of “ideal” inputs available to  $P_i$  while the  *$i^{\text{th}}$  guaranteed choice domain* in of interest in “ideal” CP worlds are the set of real world inputs available to adversary  $A_i$ .

Viewed in this light a  $\pi$ -abstraction in CP is given by the specifications of all ITMs taking part in a pair of executions whenever one CP-emulates the other. Intuitively it tells us how to obtain the same behavior from  $\mathcal{Z}$  using a different (presumably more realistic) arrangement of ITMs. Specifically, by viewing parties as dummy entities which forward their input to the resource a CP abstraction describes how to map an input  $x_i$  for (ideal world) player  $P_i$  to the “input”  $\pi_i(x_i)$ . Moreover for every input  $\bar{x}_i$  accepted by an ideal world adversary  $\text{Sim}_i(A_i)$  a CP  $\pi$ -abstraction tells us that in the real world with adversary  $A_i$  the same input can be used. Moreover the facts that:

1. The code of the  $i^{\text{th}}$  protocol depends only the interfaces available to player  $P_i$ .
2. The transformations describing the  $i^{\text{th}}$  simulator depends only on the code of the  $i^{\text{th}}$  adversary.

imply that the  $\pi$ -abstraction of CP induce a *complete factorisable relation* between choice sets satisfying Def.11 of [35]. We note that by taking this (admittedly rather informal) view of the CP framework Theorem 1 of [35] provides an alternative proof of the CP composition theorem.

**Simplified CP Security.** We note that compared to CP, SCP is even closer in spirit to the Abstract Cryptography framework. In particular AC does not distinguish between honest players

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<sup>38</sup>For example in a hybrid world with two resources  $\mathcal{F}$  and  $\mathcal{R}$  the dummy protocol  $\mathcal{D}^{\mathcal{F},\mathcal{R}}$  can be seen as the parallel composition of the protocols  $\mathcal{D}^{\mathcal{F}}$  and  $\mathcal{D}^{\mathcal{R}}$ .

and adversaries. In contrast to the flexibility (and variability) available to adversaries in the CP model, SCP essentially fixes the ITMs of all adversaries in both the ideal and real worlds bringing it one step closer to the spirit of AC.